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Major E. H. HILLS, C.M.G., D.Sc., F.R.S., PRESIDENT, in the Chair.

William Edward Curtis, A.R.C.S., D.I.C., B.Sc., Imperial College of Science, South Kensington, S.W.;
Hermann Glauert, B.A., Trinity College, Cambridge;
Major-General William du Gard Gray, C.B., 30 Alexandra Court, Queen's Gate, S.W.;
Ahmed Hussain, B.A., B.L., Chief Secretary to H.H. the Nizam's Government, Hyderabad, Deccan, India;
Rev. Thomas Laing, United Free Church Manse, Lumsden, Aberdeenshire;
Rev. Arthur Cyril Ridsdale, M.A., Howrah, Bengal, India;
Charles Robinson, M.A., 64 Rydal Road, Lancaster; and
Frank Sargent, Physics Department, Bristol University, and 16 Cottrell Road, Eastville, Bristol,

were balloted for and duly elected Fellows of the Society.

Walter S. Adams, M.A., Solar Observatory, Mount Wilson, California, U.S.A.;
Henri Andoyer, Professeur à la Sorbonne, Paris;
Julius Bauschinger, Direktor der Kaiserlichen Universitäts-Sternwarte, Strassburg; and
Frank Schlesinger, M.A., Ph.D., Director of the Allegheny Observatory, U.S.A.,

were proposed by the Council as Associates of the Society.

Seventy-one presents were announced as having been received since the last Meeting, including, amongst others :—

Maupertuis, Discours sur la parallaxe de la Lune (1741), presented by Major Hills. T. N. Orchard, Milton's Astronomy: the Astronomy of Milton's "Paradise Lost," presented by the author. Astrographic Chart; forty-eight charts from Algiers, Bordeaux, and Toulouse Observatories, presented by the French Minister of Public Instruction. A photograph of the spectrum of the nebula in Orion, presented by J. H. Reynolds.

The Determination of the Constants of the Node, the Inclination, the Earth's Ellipticity and the Obliquity of the Ecliptic from the Greenwich Meridian Observations of the Moon, 1847–1901. By Ernest W. Brown, Sc.D., F.R.S.

1. In this fourth * paper on the constants of the moon's orbit, I determine those which are found from an analysis of the coefficients of cos F, sin F, in latitude. Dr. Cowell's means over 414 days for these coefficients on which this work is based † extend only over the Hansen series, that is, from 1847 to 1901. In dealing with the mean latitude, no good reason appeared for extending the work a hundred years back, and therefore, before undertaking the heavy task of collecting and correcting the observations of the series dealt with by Airy, it seemed advisable to examine fully the Hansen series and see whether anything was likely to be gained by the extension of the work. The experience obtained in the first and third papers showed that the only possibility for increased accuracy lay in the terms or coefficients which possessed portions depending on the time, and even here the improvement was doubtful owing to the large errors with which the earlier observations are affected.

For comparison, the results at which Newcomb arrived in his posthumous memoir are available. An unexpected agreement was found in all cases, more particularly in that constant which might have been improved with a longer series of observations, namely, the mean motion of the node. The longitude of the node at epoch differs by a quantity which is very little greater than its probable error; the other differences are less than the probable

* The previous papers deal with the mean longitude (*M.N.*, vol. lxxiii., No. 9), the mean latitude (*ib.*, lxxiv., No. 3), and the perigee and eccentricity (*ib.*, No. 5).

† *M.N.*, vol. lxv. p. 740.

errors. The systematic errors of the Airy series make its use for the determination of the earth's ellipticity and the obliquity of the ecliptic of doubtful value: the errors are not well enough known. At the present time, therefore, I see no sufficient reason for undertaking the extra work, so far as the determination of constants for the tables of the moon is concerned.

2. Newcomb laid some stress on his determination of the earth's ellipticity and the secular change of the obliquity from observations of the moon. The observed coefficients, on which these constants principally depend and which are deduced below from the meridian observations, agree very closely with his. In the years which elapsed between the completion and appearance of Newcomb's paper, a new value for the ellipticity was obtained by Mr. Crommelin in his discussion of the last Cape-Greenwich determination of the moon's parallax. This value is close to that which I have found from the residuals of the mean motions of the perigee and node; it is therefore close to Clarke's value, and differs considerably from Helmert's determination and from the value adopted at the Conference of Nautical Almanac directors in 1911. In view of these agreements and disagreements, a new direct determination is desirable over a different meridian: the Harvard photographic method applied, if possible, to Mösting A would seem to promise more accurate results than any other.

The periodic term in latitude which depends on this constant also has a portion depending on the obliquity of the ecliptic. Hence, assuming that the value of the ellipticity obtained from the perigee and node and direct observation is correct, we should be able to obtain the actual obliquity as well as its secular variation. Newcomb thought that the value which he obtained from occultations for the latter was as accurate as any yet found from observations of the sun; his result is confirmed by the Greenwich observations in so far as this confirmation is worth anything. He made no attempt to find the obliquity at epoch: on the same basis, the value for the latter obtained below should be as accurate as any yet found.

3. The terms attributed to the systematic irregularities of the moon's limb at the end of my paper on the mean latitude are here considered in somewhat greater detail. This origin for the terms appears to be well founded and to constitute a step forward in removing empirical terms from the moon's motion. Since the establishment of the existence of such terms from the observations is a first requirement, I have given the analysis and the methods used with more detail than is usual in finding periodic terms by the Fourier analysis.*

* Since this work was finished, papers by J. E. de Vos van Steenwijk and E. F. van de Sande Bakhuysen have appeared in the *Proceedings* of the Kon. Akad. van Wetens. te Amsterdam. My conclusions as to the source of these inequalities have been anticipated by Professor Bakhuysen. As the work in my papers was quite independent, I leave it unchanged. The confirmation afforded by the two investigations is, I think, conclusive (February 15).

Tabular Corrections.

4. A comparison of Cowell's transformation * of Hansen's tabular theory with my theory gave the following corrections to be applied to the tabular-observed residuals depending on cos F, sin F :—

$$+0''.02 \sin (F + 2D - 2F + 2T - 2J)$$
$$+0''.04 \sin (F + 2D - 2F - 3T + 3V)$$
$$+0''.07 \sin (F + 2D - 2F - 6T + 5V + 270^\circ)$$
$$+0''.08 \sin (F + \Omega + 5T - 3V + 216^\circ)$$
$$+0''.07 \sin (F + \Omega - 5T + 3V + 52^\circ)$$
$$+0''.03 \sin (F + \Omega + 8T - 5V + 125^\circ)$$
$$+0''.08 \sin (F + 2\Omega)$$
$$+0''.06 \sin (\Omega + 142^\circ) \cos F,$$

which were tabulated in units of $0''.01$ as coefficients of cos F, sin F by Mr. Hedrick. The last term is obtained from the comparison of Hansen's node terms with mine.†

The coefficient of cos F also requires the corrections $2\gamma\delta L$, $-2\gamma\delta\Omega$, where δL, $\delta\Omega$ are the corrections to be applied to the mean longitude and the node. These are

$$\delta L = \text{corrections in No. 13 of my longitude paper } \ddagger$$
$$+0''.24 \sin (13T - 8V + 314^\circ),$$
$$-\delta\Omega = -0''.63 \sin \{20^\circ.2(t_c - 0.5) + 41^\circ.1\} \S$$
$$-0''.18 \sin (13T - 8V + 314^\circ) \S$$
$$-0''.17 \sin (l + 16T - 18V + 151^\circ).$$

These corrections were tabulated to $0''.01$ and then multiplied by $.0895$. Both sets of corrections are included in the second columns of Tables II., III.

5. *The correction for empirical terms in the mean longitude.*—The existence of an empirical term or terms in the mean longitude which can scarcely arise from errors of observation has been demonstrated in my longitude paper. These, multiplied by 2γ, should be added to the coefficient of cos F, so that any remaining terms will be strictly perturbations of the node.‖ The fifth column of Table II. therefore contains $-.0895\ \delta L$, where δL consists of the tabular minus observed residuals in the last column of Table II. of my longitude paper.

6. *Allied terms.*—No new sensible allied terms beyond those

* *M.N.*, vol. lxiv. p. 165.

† *Ibid.*, vol. lxxiv., No. 5.

‡ *Ibid.*, vol. lxxiii. pp. 700, 701.

§ See errata to part v. of my theory of the moon's motion in *M.N.*, vol. lxxiv. p. 424.

‖ See No. 14 of my paper on the perigee and eccentricity.

mentioned by Dr. Cowell* were found. My corrections for these are

$$+0''\cdot039 \sin (F+D) - 0''\cdot097 \sin (F-D),$$

producing a tabular minus observed correction, $+0''\cdot029 \sin F$. This was not applied but is included in the discussion of the inclination in No. 21.

Observational Corrections.

7. The investigation of these corrections, with the exception of the obliquity, has been given in my papers on the mean longitude and on the mean latitudes of the moon and sun. We need here, however, the periodic corrections which depend on the right ascension, that is, on the angle $F+\Omega$, with sufficient approximation. The term due to the earth's ellipticity has the same period, and consequently it was necessary to find them with all the accuracy possible.

8. *Corrections to the declinations.*—The method of obtaining them is explained in Nos. 6, 7 of my mean latitude paper. The portions needed here are of the form $b \sin \alpha$, where α is the right ascension. The following table gives the adopted coefficients b:—

TABLE I.

Corrections to Observed N.P.D. Coefficients of sin α. Units of 0″·01.

Year.	+0.	+10.	+20.	+30.	+40.	+50.	+60.
1841	...	+20	+15	−46	+23	+19	+17
1842	...	+20	+37	−43	+19	+20	...
1843	...	+19	+39	−44	+21	+24	...
1844	...	+17	+35	−43	+21	+20	...
1845	...	+20	+45	−40	+23	+16	...
1846	...	+17	−10	−41	+22	+15	...
1847	+21	+18	− 8	−38	+22	+ 9	...
1848	+21	+21	−55	− 5	+25	+ 9	...
1849	+21	+14	−55	0	+23	+11	...
1850	+21	+21	−52	− 3	+20	+14	...

9. *Corrections for equinox.*—An error $\delta\alpha$ of the assumed equinox produces an error $\sin \epsilon . \cos \alpha \, \delta\alpha$ in the E.N.P.D. The adopted errors of the equinox are given the last column of the table in No. 12 of my longitude paper. These multiplied by ·4 cos α give the corrections to be applied.

10. *Parallax correction.*—This is found in No. 9 of my mean latitude paper. The periodic part is

$$1847\text{–}1861, +0''\cdot02 \sin \alpha; \quad 1862\text{–}1901, +0''\cdot09 \sin \alpha.$$

11. *Correction to the obliquity of the ecliptic.*—If $\delta\epsilon$ be this correction, $\delta\epsilon \sin \alpha$ must be applied to the observed E.N.P.D.

* *M.N.*, vol lxv. p. 564

According to Newcomb,* the obliquity of Bessel's Tabulæ Regiomontanæ was used until 1862, that of Leverrier from 1863 to 1900, and his own value after that date. The reductions to his own value given by Newcomb and adopted here are

$$1847\text{–}1862,\quad \delta\epsilon = -0''\cdot27 - 1''\cdot14(t_c - 0\cdot5),$$
$$1863\text{–}1900,\quad \delta\epsilon = -0''\cdot15 + 0''\cdot75(t_c - 0\cdot5),$$

where t_c is reckoned in centuries from 1800.

12. The corrections for declination, equinox, and parallax were added together and tabulated as corrections to the coefficients of cos F, sin F; they have the node as argument. They are contained in the third columns of Tables II., III. The next following column in each case contains the obliquity correction treated in a similar manner.

The Final Residuals.

13. These are given in the following tables for cos F, sin F respectively. The first column in each table gives the period, the next three columns contain the tabular and observational corrections, and the fifth column in Table II. the correction for the shorter period empirical terms in the mean longitude, all in units of $0''\cdot01$. The columns with heading C. contain Cowell's residuals,† those with heading Br. contain the sum of the preceding columns and therefore the residuals ready for analysis. The final columns, headed Res., contain the same residuals after the removal of the term $0''\cdot35 \cos(F+\Omega+8^\circ)$ and, in the case of cos F, the secular part, $+0''\cdot30 - 0''\cdot0100\,P$ also. The last three columns are given in units of $0''\cdot1$.

TABLE II.

Coefficients of cos F. Tabular – observed Residuals.

	Units of $0''\cdot01$.				Units of $0''\cdot1$.		
Per.	Tab.	δ,a,π.	Obl.	$2\gamma\delta L$.	C.	Br.	Res.
86	− 2	−51	+ 5	+ 8	− 6	−10	− 4
87	− 6	−36	− 4	+18	− 7	−10	− 4
88	− 7	0	−14	+12	+ 5	+ 4	+10
89	−10	+10	−23	+19	− 5	− 5	0
90	−15	+17	−29	+24	− 4	− 4	− 1
91	−18	+41	−31	+22	− 2	− 1	+ 1
92	− 3	+24	−28	+24	− 3	− 1	0
93	+10	+24	−21	+27	− 3	+ 1	+ 1

* *Amer. Eph. Papers*, vol. ix. part i. p. 47; I have not been able to verify this statement but suppose that Newcomb had found out that such was the case. A search through the introductions to most of the Nautical Almanacs issued since Hansen's tables were introduced makes the matter doubtful. The earlier volumes state that the right ascension and declination were obtained from Hansen's tables, which contain a value of the obliquity different from that of Leverrier; the later volumes make the statement only with reference to the latitude and longitude. The American Ephemeris is equally indefinite.

† *M.N.*, vol. lxv. p. 740.

TABLE II.—*Coefficients of cos F. Tabular – observed Residuals*—continued.

	Units of 0″·01.				Units of 0″·1.		
Per.	Tab.	δ,α,π.	Obl.	$2\gamma\delta$L.	C.	Br.	Res.
94	+ 1	+11	−10	+27	− 3	0	− 1
95	− 9	+ 3	+ 3	+25	− 1	+ 1	0
96	+ 1	− 5	+17	+25	+ 1	+ 5	+ 4
97	+11	−12	+29	+27	− 5	+ 1	0
98	+14	−16	+38	+28	−11	− 5	− 5
99	+22	−42	+41	+31	− 2	+ 3	+ 5
100	+26	−55	+ 5	+34	− 3	− 2	0
101	+21	−41	+ 3	+29	− 3	− 2	+ 1
102	+15	−27	+ 1	+24	− 5	− 4	0
103	+16	−14	0	+18	− 8	− 6	− 2
104	+ 5	−22	0	+13	− 6	− 6	− 2
105	−11	−43	− 1	+11	0	− 4	0
106	−10	−47	0	+ 7	− 3	− 8	− 5
107	+10	−38	+ 1	− 1	− 3	− 6	− 4
108	+29	−25	+ 2	− 4	0	0	0
109	+34	−12	+ 2	−11	0	+ 1	0
110	+26	+ 4	+ 2	−11	0	+ 2	− 1
111	+27	+18	+ 3	−15	− 5	− 2	− 5
112	+38	+29	− 2	−21	+ 1	+ 5	+ 2
113	+37	+19	− 4	−25	0	+ 3	0
114	+26	+ 1	− 6	−18	+ 9	+ 9	+ 7
115	+26	− 4	− 8	−22	+ 3	+ 2	+ 1
116	+29	−36	− 9	−20	+ 1	− 3	− 3
117	+27	−35	− 8	−22	+ 1	− 3	− 1
118	+27	−31	− 6	−20	+ 3	0	+ 3
119	+23	−22	− 2	−22	− 3	− 5	− 2
120	+ 5	−10	+ 3	−28	+ 4	+ 1	+ 3
121	− 3	+ 3	+ 8	−25	+ 5	+ 3	+ 5
122	+16	+19	+12	−26	+ 2	+ 4	+ 5
123	+33	+27	+15	−23	+ 4	+ 9	+ 9
124	+31	+32	+15	−21	0	+ 6	+ 5
125	+26	+36	+13	−26	+ 1	+ 6	+ 4
126	+29	+39	+10	−33	− 2	+ 3	− 1
127	+37	+36	+ 4	−30	− 3	+ 2	− 2
128	+31	+26	− 3	−28	+ 1	+ 4	0
129	+15	+14	−10	−28	0	− 1	− 5
130	+ 3	+ 4	−17	−26	+ 6	+ 2	− 2
131	+14	− 9	−21	−19	+ 9	+ 6	+ 3
132	+27	−25	−23	−14	+ 1	− 3	− 5
133	+31	−36	−12	−11	− 1	− 4	− 5

TABLE III.

Coefficients of sin F. Tabular – observed Residuals.

	Units of 0″·01.			Units of 0″·1.		
Per.	Tab.	δ,α,π.	Obl.	C.	Br.	Res.
86	+ 7	− 13	+ 24	− 4	− 2	− 3
87	0	− 30	+ 25	− 6	− 7	− 7
88	− 5	− 27	+ 23	+ 4	+ 3	+ 4
89	− 4	− 24	+ 17	+ 2	+ 1	+ 4
90	− 7	− 19	+ 7	− 1	− 3	0
91	− 14	− 11	− 4	− 5	− 8	− 5
92	− 15	− 2	− 16	+ 1	− 2	+ 1
93	+ 5	+ 9	− 26	0	− 1	+ 2
94	+ 26	+ 17	− 34	− 2	− 1	0
95	+ 23	+ 21	− 36	− 2	− 1	− 1
96	+ 12	+ 20	− 33	+ 1	+ 1	0
97	+ 10	+ 17	− 26	+ 3	+ 3	+ 1
98	+ 9	+ 11	− 13	+ 6	+ 7	+ 4
99	0	+ 20	+ 2	+ 7	+ 9	+ 6
100	+ 3	− 4	+ 2	+ 5	+ 5	+ 2
101	+ 3	− 24	+ 3	+ 3	+ 1	− 2
102	− 1	− 30	+ 3	+ 3	0	− 2
103	+ 1	0	+ 2	− 2	− 2	− 2
104	+ 13	+ 17	+ 1	− 5	− 2	− 1
105	+ 9	+ 23	+ 1	+ 2	+ 5	+ 7
106	− 5	0	0	− 2	− 3	0
107	− 8	− 19	0	− 3	− 6	− 3
108	+ 2	− 31	+ 1	− 7	− 10	− 7
109	+ 18	− 39	+ 2	− 7	− 9	− 6
110	+ 19	− 38	+ 4	− 1	− 3	− 1
111	+ 4	− 32	+ 5	+ 4	+ 2	+ 3
112	− 4	− 21	+ 6	− 2	− 4	− 5
113	− 1	+ 7	+ 5	0	+ 1	− 1
114	− 9	+ 20	+ 3	+ 3	+ 4	+ 1
115	− 20	+ 19	+ 1	+ 4	+ 4	+ 1
116	− 20	+ 8	− 3	+ 1	− 1	− 4
117	− 15	− 5	− 6	+ 4	+ 1	− 2
118	− 8	− 17	− 10	+ 3	− 1	− 3
119	+ 9	− 28	− 11	− 1	− 4	− 5
120	+ 16	− 34	− 12	+ 2	− 1	− 1
121	− 2	− 36	− 11	+ 5	0	+ 2
122	− 14	− 33	− 7	+ 5	0	+ 3
123	− 8	− 27	− 3	− 1	− 5	− 2

TABLE III.—*Coefficients of sin F. Tabular – observed Residuals*—continued.

	Units of 0″·01.			Units of 0″·1.		
Per.	Tab.	δ,α,π.	Obl.	C.	Br.	Res.
124	− 1	−16	+ 3	− 2	− 3	0
125	− 6	− 3	+ 9	− 2	− 2	+ 1
126	−15	+10	+14	− 1	0	+ 2
127	−14	+23	+17	− 5	− 2	− 1
128	− 1	+31	+19	−12	− 7	− 7
129	+ 3	+37	+17	− 9	− 3	− 4
130	− 8	+35	+13	− 3	+ 1	− 1
131	−16	+34	+ 6	0	+ 2	− 1
132	− 8	+29	− 3	+ 2	+ 4	+ 1
133	+ 4	+16	− 5	0	+ 2	− 1

Analysis of the Residuals.

14. The subjects for analysis are the numbers in columns Br. of Tables II., III. Two approximations for the node term and for the secular part of the coefficient of cos F were made, the second, from the columns Res., being carried out after the removal of the terms produced by the first. What is given below is the second approximation.

The analysis for the term with the node as argument was made by performing it with a motion of − 22°·5 in a period instead of with − 21°·93, the motion of the node. There are then exactly three cycles in the 48 periods at our disposal. The phase and amplitude in each cycle being obtained, the value of Ω + half the difference of its motion from − 22°·5 was subtracted from each phase; the resulting phases and amplitudes then give immediately the coefficients of cos Ω and sin Ω.

After the removal of the node term from the coefficient of cos F, the analysis for terms of the form $a + bt$ was made on sums of four in the usual manner.

15. *Secular parts.*—The sums of four residuals in units of 0″·1 in the coefficient of cos F, after the removal of the Ω term and the terms, $+0''\cdot72 - 0''\cdot88t_c$, are

$$+2,\ +1,\ +3,\ +1,\ -4,\ -9,\ -4,\ +4,\ +9,\ +23,\ -8,\ -9.$$

There is evidence of a term of long period with which I shall deal later (No. 18). The solution by least squares gave, after division of the result by ·0895 (the coefficient of the principal term in latitude), the correction to be applied to the adopted value of F,

$$+8''\cdot3 - 10''\cdot0t_c,$$

where t_c is reckoned as usual from 1800, with a probable error of about 1″ in each coefficient.

16. The means of 16 residuals of the coefficients of sin F as they stand in the column Res. of Table III. are $+0''\cdot04$, $-0''\cdot14$, $-0''\cdot11$, giving a mean correction, $+0''\cdot07$, to the coefficient

of sin F in latitude, with a probable error of $\pm 0''.04$. The result was the same in the first approximation.

17. *The terms with argument* Ω.—These were obtained in three cycles as the coefficients of products of cos F, sin F with cos Ω, sin Ω. The transformation into four periodic terms gave for the three cycles and for their means—

	I.	II.	III.	Mean.
	"	"	"	"
cos (F + Ω)	+0·44	+0·44	+0·24	+0·37
sin (F + Ω)	−0·01	−0·18	+0·15	−0·01
cos (F − Ω)	+0·00	+0·08	+0·04	+0·04
sin (F − Ω)	+0·03	+0·06	−0·24	−0·05

The probable error, as usual, is about $0''.04$ in each case. As the first two of these terms are important, I made still another approximation after the removal of the doubtful 28 P term (No. 18) in cos F, with the following result—

	I.	II.	III.	Mean.
	"	"	"	"
cos (F + Ω)	+0·40	+0·44	+0·30	+0·38
sin (F + Ω)	−0·04	−0·11	+0·06	−0·03
cos (F − Ω)	−0·04	+0·08	+0·10	+0·05
sin (F − Ω)	+0·07	−0·04	−0·15	−0·04

The discordances between the three cycles are somewhat less; the mean is scarcely altered as might have been expected, since there are very nearly two cycles of the 28 P term within the period.

If, with Newcomb, one attempts to find the secular change of the obliquity from the variation of the coefficient of sin (F + Ω), that is, if the coefficients be analysed for a term of the form $a + bt$, the results in the two cases are,

$$-0''.12 + 0''.44(t_c - .5), \quad -0''.10 + 0''.28(t_c - .5),$$

the probable error of the coefficient of t_c being $0''.2$.

A similar process with the coefficient of cos (F + Ω) gives

$$+0''.51 - 0''.55(t_c - .5), \quad +0''.45 - 0''.28(t_c - .5).$$

18. *The term of period* 28 P.—The analysis was performed with a motion of 30° in 2P, sums of two residuals being used. There are two cycles. Those for cos F gave

$$+0''.17 \sin (15°t + 340°), \quad +0''.34 \sin (15°t + 294°),$$

t being the number of periods from P = 86·5, 110·5, respectively. The indicated motion of the argument is − 46° in 24 P. The term is therefore

$$+0''.23 \sin (13°t + 16°),$$

where t is the number of periods from 1850. The term so obtained has a period of about 31 years.

If the same process be followed with the coefficients of sin F, the results are

$$+0''\cdot23 \sin (15^\circ t + 289^\circ), \quad +0''\cdot03 \sin (15^\circ t + 225^\circ).$$

The period cannot be found from this.

Both terms are quite doubtful. That in sin F should, I think, be treated as wholly accidental. That in cos F may possibly exist, but the material is too scanty for any definite statement. If it is real, the coefficient will be about $0''\cdot2$ and the period between 25 and 35 years.

19. There are indications of terms of shorter period, more particularly in the coefficient of sin F. It does not seem profitable to analyse for these at the present time. Even if they are real, no effect would be produced on the mean results obtained in the discussion which follows.

Discussion of the Results.

20. All the results obtained above are deduced from tabular minus observed residuals. Hence, for application as corrections to the constants of the theory, they must have their signs changed.

The Inclination.

21. The coefficient of the principal term in latitude is determined as follows:—

Hansen's coefficient*	18461″·65	
Addition by Cowell†	−0·30	
Correction for allied terms (No. 6) .	−0·03	
Analysed correction (No. 16) . .	+0·07	±0″·03
Observed coefficient	18461″·39	±0″·03

The addition by Cowell is one which he made to all the residuals adopted here before setting them down. He found from the same observations $18461''\cdot35 \pm 0''\cdot05$. The small difference is undoubtedly due to the corrections, chiefly observational, which have been made above.

Newcomb‡ obtained a correction $-0''\cdot19 \pm 0''\cdot19$ to Hansen's coefficient. I estimate the correction for allied terms as somewhat less than that for meridian observations, say $-0''\cdot02$. His observed coefficient is then $18461''\cdot44$. Owing to the large probable error,

* *M.N.*, vol. lxiv. p. 165. In the comparison of my coefficients in latitude with those of Hansen (*M.N.*, vol. lxv. p. 286), the coefficient quoted is erroneous as I conjectured. The differences there marked (*f*) consequently disappear.

† *Ibid.*, lxv. p. 744.

‡ *Amer. Eph. Papers*, vol. ix. part i. pp. 223, 219.

it is not entitled to great weight in spite of the longer series. It will be convenient to adopt a round number,

$$18461''\cdot400,$$

as the required constant.

The Node.

22. The results of the investigation are given in the following summary:—

F. (Hansen).	. $302°26'55''\cdot55+(1342^r+$	$82°02'39''\cdot22)t_c+5''\cdot11t_c^2$
Cor. (Newcomb) .	$-1''\cdot14$	$-29''\cdot17t_c-3''\cdot76t_c^2$
L_0 . . .	$335°43'25''\cdot56+(1336^r+307°53'12''\cdot37)t_c+9''\cdot52t_c^2$	
Ω, adopted . .	$33°16'31''\cdot15-$	$(5^r+134°08'57''\cdot68)t_c+8''\cdot17t_c^2$
Addn. (Cowell) .	$+5''\cdot59$	
Theor. cor. . .	$-0''\cdot18$	$+0''\cdot70t_c-0''\cdot70t^{c2}$
Anal. cor. (No. 15)	$-8''\cdot3$	$+10''\cdot0t_c$
Ω, observed . .	$33°16'28''\cdot26-$	$(5^r+134°08'46''\cdot98)t_c+7''\cdot47t_c^2.$

The first line is Hansen's tabular $g+\omega$. The second is Newcomb's correction applied in the *Nautical Alamanac* since 1882 to all arguments containing the mean longitude; all the observations from 1847 to that date were corrected by Cowell for this change. The third is the adopted mean longitude (No. 4) to which all the residuals have been corrected above. The fourth line is L_0 less the sum of the two previous lines and therefore the adopted Ω. The fifth line is $+0''\cdot50\div\cdot0895$, due to the addition by Cowell* of $0''\cdot5$ to all the residuals of the coefficient of cos F.

The next line contains the correction to the secular acceleration necessary to reduce it to its theoretical value. It is of course impossible to obtain a correction from observations extending only over 54 years. The only question at issue is the epoch at which it should be made. The mean epoch of the observations used here is 1875, but in the case of the perigee it was 1850, and this date is also more suitable for comparisons in general. My theoretical value is $+6''\cdot36t_c^2$, to which $1''\cdot11t_c^2$ is to be added for the acceleration of the equinox. The correction is therefore $+0''\cdot70t_c^2$ at 1850 and hence the value set down at 1800, this being the date from which t_c is reckoned.

The final line, the sum of the four preceding lines, of which the last is the analysed correction (No. 15), gives the value of the longitude of the node as deduced from the Greenwich observations.

23. In order to compare Newcomb's last value† for this quantity with mine, some small corrections must be made. One, for a planetary term of very long period which he has omitted, is $-0''\cdot32-0''\cdot17t_c$. The others, for a slight difference in his adopted theoretical value and for the change of Hansen's value to the theoretical value at 1850 (which he has made at 1800), con-

* *M.N.*, vol. xlv. p. 744. † *Loc. cit.*, p. 224.

tribute $-0''\cdot18+0''\cdot70t_c-0''\cdot09t_c^2$. With these corrections his value becomes

$$\Omega=33^\circ16'26''\cdot18-(5^r+134^\circ8'46''\cdot90)t_c+7''\cdot47t_c^2,$$

with a probable error, $\pm0''\cdot7\pm1''\cdot7t_c$. The difference is

$$\text{Brown-Newcomb}=+2''\cdot1-0''\cdot1t_c.$$

The mean of the two results is not much greater than its probable error. I adopt

$$\Omega=33^\circ16'27''\cdot50-(5^r+134^\circ8'47''\cdot00)t_c+7''\cdot47t_c^2.$$

The term t_c^3 is omitted for reasons stated in No. 25 of my paper on the perigee and eccentricity.

25. The agreement of the value for the mean motion of the node deduced from the occultations and the meridian observations is particularly satisfactory, the more so since there is a well-marked difference from my theoretical value. The latter as published * requires a correction $+3''t_c$ to reduce the portion depending on the earth's ellipticity to the value 1/297 adopted here. The epoch is 1850. Transforming the observed value to the same epoch, I find

Observed value . . .	$-6967945''(t_c-\cdot5)$,
Theoretical value . . .	$-6967933(t_c-\cdot5)$,
Obs. – theor. . . .	$-12(t_c-\cdot5)$,

tenths of a second being omitted because the theoretical value was only obtained to the nearest second.

This discordance involves a difference ranging from $0''\cdot0$ to $0''\cdot5$ in the coefficient of cos F during the interval 1850 to 1900. Any such large mean correction seems to be out of the question. It appears then that we must adopt the observed value and attribute the difference to inaccurate values of the ellipticities of the earth and moon (which are uncertain to at least this extent), to unknown perturbations, or to a combination of all three. With respect to the ellipticity of the moon's figure, I believe that a correction on this account is more likely to increase the discordance than diminish it: the adopted constant being determined from librations, irregularities of the limb do not arise in this connection.

The Obliquity of the Ecliptic and the Earth's Ellipticity.

26. An error $\delta\epsilon_0+t\delta\epsilon_1$ in the adopted obliquity produces a tabular error, $(\delta\epsilon_0+t\delta\epsilon_1)$ sin a, in the latitude of the moon.

An error δk in the inverse, k, of the earth's ellipticity produces a tabular error, $-(2C\delta k/k)$ sin a, in the term C sin a in latitude.

Errors in the declination system and in the parallax also produce terms of the form a sin a, but the latter may be neglected since it could hardly amount to more than two or three hundredths of a second of arc.

Hence, as far as the constant part of the coefficient of sin a is

* *Mem. R.A.S.*, vol. lix. p. 94.

concerned, at least three causes may be assigned to any residual found by observation.

On the other hand, the portion of the coefficient which varies with the time cannot well be assigned to anything but an error in the secular change of the obliquity, so that we may determine the latter without ambiguity.* It is of course possible that there may be a secular change of the inclination of the moon's mean orbit, but the general theory of perturbations makes doubtful any ordinary gravitational source for this.

27. The adopted coefficient of $\sin a$ is $-8''\cdot312$. Adding the corrections obtained in No. 17, I find, for the observed coefficient,

$$-8''\cdot19 - 0''\cdot44(t_c - \cdot5), \quad -8''\cdot21 - 0''\cdot28(t_c - \cdot5),$$

according as we exclude or include the 28 l' term. At the mean epoch 1875, these become $-8''\cdot30$, $-8''\cdot28$, respectively.

Newcomb's correction to his adopted coefficient,† $-8''\cdot271$, is $+0''\cdot09 - 0''\cdot40\,(t_c - \cdot5)$, making his observed value,

$$-8''\cdot18 - 0''\cdot40(t_c - \cdot5),$$

with probable error, $\pm 0''\cdot07 \pm 0''\cdot16\ (t_c - \cdot5)$; this agrees remarkably well with my values. The mean epoch of Newcomb's determination is 1869, so that, if the variable part of the coefficient had been neglected, the coefficient would have been $-8''\cdot26$.

In spite of the agreement of both variable and constant parts, the former must be accepted with caution on account of the large probable error $0''\cdot2$ in each determination. It must be added also that the coefficients of $\cos(F - \Omega)$, $\sin(F - \Omega)$ can be analysed (No. 17) into variable portions of the same order of magnitude, and there does not seem to be any cause for such variation. In fact, the agreement between the two determinations—and this may be accidental—is the only real foundation for any extended discussion.

28. Newcomb considers (*loc. cit.*, p. 226) that his determination from the occultations is the most reliable of any hitherto made. As the evidence for the reality of the correction is strengthened by the Greenwich meridian observations, I give below the principal determinations which have been made for the secular change of the obliquity per century at the epoch 1850:—

Hansen	$-46''\cdot78$
Bessel	$-45\cdot7$
Leverrier	$-47\cdot59$
Newcomb	$-46\cdot84$
,, from occultations .	$-47\cdot24$
Gr. mer. obs. of the moon .	$-47\cdot28$
	$-47\cdot12$

the last two corresponding to the two values obtained above.

* See Newcomb, *loc cit.*, p. 226.

† This is Hansen's value which Newcomb states erroneously on p. 225 of *Amer. Eph. Pap.*, vol. ix. part i. to be $-8''\cdot63$.

29. The correction to the value at epoch deduced from the residuals in the motion of the moon is more doubtful in view of the facts stated above. The whole matter may be briefly stated in the following way, as far as observations of the moon are concerned.

The observations give a coefficient to sin a of

$$-8''\cdot19 - 0''\cdot40(t_c - \cdot5),$$

with probable error, $\pm 0''\cdot04 \pm 0''\cdot14\ (t_c - \cdot5)$, with Newcomb's value of the obliquity, his system of declinations, and the ellipticity 1/297. Whatever changes are made in the constant parts of these, either or both of the others must be so changed that the observed coefficient remains the same.

One result of the discussion is very satisfactory, namely, the determination by observation of this coefficient within $0''\cdot04$, at the mean epoch. Many values have been given differing widely from one another: Hansen found $-8''\cdot38$ and Faye $-8''\cdot59$. Cowell's value is $-8''\cdot14$. This last was found from the same observations as my value, and the various corrections which I have made have changed it very little.

The Ellipticity of the Earth.

30. Newcomb (*loc. cit.*, p. 225) attempts to determine the ellipticity of the earth from the observed differences of the motions of the perigee and node from their theoretical values. I am not certain whether such a determination standing alone is of great value, but as Newcomb also used the coefficient of sin a in latitude for the same purpose (erroneously, as stated above), it may be useful to give a summary of the evidence. The determination of the perigee from the Greenwich observations is quoted from my paper on the perigee and eccentricity.

Whether we take into account a change of the obliquity or not, the mean value of the coefficient of sin a is $-8''\cdot28$ (1875), the three values considered not differing from this by more than $0''\cdot02$.

31. The ratios of the residuals of the mean motions of the perigee and the node and of this coefficient to the portions of them depending on the ellipticity are

$$\frac{+17''}{+638''},\ \frac{-12''}{-597''},\ \frac{-0''\cdot03}{-8''},$$

or $+\cdot027$, $+\cdot020$, $+\cdot004$, with probable errors of 2, 2, 5 units in the last places respectively. As the last includes the correction to the obliquity, I deduce the correction to the ellipticity from the mean of the first two; the resulting correction to the obliquity deduced from the third will then be $+0''\cdot15$. The corrected ellipticity is given by

$$\frac{1}{k} = 293\cdot5 \pm \cdot3.$$

The last Cape-Greenwich determination of the parallax * gave, by comparison of the observed with the gravitational value,

$$\frac{1}{k} = 294{\cdot}4 \pm 1{\cdot}5.$$

The two results are in good agreement. The possible error of the former, however, is not to be judged from the probable error set down, owing to the uncertainty of the portions in the mean motions of the perigee and node due to the ellipticities of the moon. The two results should, I believe, be given about equal weight, so that the deduced value would be $1/k = 294{\cdot}0$.

The mechanical ellipticity, on which the motions of the perigee and node depend, is deduced from the surface ellipticity by the usual formula, while the Cape-Greenwich value depends directly on the surface ellipticity. It is somewhat remarkable that the two determinations should agree so closely, more particularly in view of the fact that the latest values have tended to a smaller result. Those quoted by Newcomb are:—Bessel 299·15, Listing 288·48, Clarke 293·46, Helmert 298·26, the Conference value being 297·0.

The value $1/k = 294{\cdot}0$ requires an addition to the term in longitude with the node as argument of $+0''{\cdot}15$. There is no difficulty in this since the results in my longitude paper showed that a determination of this coefficient from observation must be uncertain by at least this amount owing to the empirical terms and the large observational errors in the Airy series.

The Obliquity of the Ecliptic.

32. If the value $1/k = 294{\cdot}0$ be accepted and it be assumed that Newcomb's declination system is correct, at least so far as the portions which vary with the declination are concerned, we must add to Newcomb's solar value† of the obliquity, $+0''{\cdot}13$ which, together with the correction previously investigated, makes a total correction, $+0''{\cdot}23 - 0''{\cdot}40\ (t_c - {\cdot}5)$. The obliquity is therefore

$$23^\circ 27' 31''{\cdot}91 - 47''{\cdot}24(t_c - {\cdot}5).$$

This new value differs from that of Leverrier by $-0''{\cdot}08$ at 1850 and by $+0''{\cdot}09$ at 1900.

The Systematic Irregularities of the Moon's Limb.

33. *The term,* $-0''{\cdot}37\ \cos a$.—This is a well-marked term in the latitude for which no theoretical cause can be assigned. The secular part of its coefficient which has been tentatively found in No. 17 is $+0''{\cdot}55t_c$ or $+0''{\cdot}028t_c$. Newcomb finds $-0''{\cdot}21t_c$ with a probable error $\pm 0''{\cdot}30t_c$, which he attributes to an error of the equinox.‡ The large probable errors make the further discussion

* *M.N.*, vol. lxxi. (1911), p. 540. † *Loc. cit.*, p. 47.
‡ *Loc. cit.*, p. 219.

of this portion unprofitable. For the constant part of the coefficient Newcomb gets $+0''\cdot15 \pm 0''\cdot12$ at 1850, which may in his work be attributed to an error of the equinox, but which may also arise from irregularities of the moon's limb. I therefore leave it aside.

In my papers on the mean latitude and on the perigee and eccentricity, I have found the following terms, to which are added that just mentioned and certain terms derived by Newcomb from the occultations:—

In Latitude:	Mer. Obs. . . .	$-0''\cdot51 - 0''\cdot37 \cos \alpha$.
	Occ. Imm. . . .	$-0''\cdot53$
	Occ. Emer. . . .	$-0''\cdot15$
In Longitude:	Mer. Obs. . . .	$-0''\cdot50 \cos \alpha$.

34. I have indicated at the end of my mean latitude paper what I believe to be the origin of these terms, and have described the shape of the moon's limb which would qualitatively fit the constant terms in latitude. The original description was, in fact, drafted before I was able to obtain Franz's figure there referred to.

It was seen that the constant term, $-0''\cdot51$, places the centre of the observed polar axis half a second below the centre of mass; that the term, $-0''\cdot53$, means a large average depression of the N.E. to E. portion as compared with the S.E. to E. portion; and that the term, $-0''\cdot15$, means a small average depression of the N.W. to W. portion as compared with the S.W. to W. portion. On the whole, therefore, the observed centres of most of the diameters through the centre of mass lie south of the equator.

Terms depending on cos α obtained from the meridian observations would then arise from the varying inclination of the polar axis to the meridian on which the moon is observed—on the average from $+23°$ to $-23°$—if the limbs are not symmetrical about this axis: the signs of the coefficients of both of the terms in cos α agree with the description of the limbs just given. The effect is, however, mixed with the personal equation of the observer, who would be apt to make the same kind of setting on a particular part of the limb each time that part happened to be on the meridian. In the meridian observations of longitude as well as in those of latitude the effects from opposite limbs are averaged, so that the actual differences are undoubtedly considerably greater in one of the limbs in each case than those exhibited by the coefficients of cos α. To find them, it would be necessary to separate north from south limb observations and east from west. To obtain quantitative comparisons an extensive integration of the inequalities measured directly over the portions of the limbs which come on to the meridian would be necessary.

In Franz's figure the axis roughly dividing the low from the high ground is near the moon's equator, the high ground being in the southern half. This may be compared with the land and water hemispheres into which the earth's surface may be roughly divided, as is well known. The distance between the centres of figure and

mass has also an interesting bearing on the question of isostatic compensation: according to this hypothesis, the density near (within 50 or 100 miles) the surface in the lower half must be on the average less than that in the upper half.

35. Are these terms to be inserted in the tables of the moon's motion? If so, corrections depending on the selenographical latitude will have to be made, and therefore they will differ with different classes of observations. It would seem better that the ephemerides should give the position of the moon's centre of mass and that the observations be corrected from a double-entry table with the right ascension and selenographical latitude, east and west, as arguments. But this table cannot be properly made until the corrections for opposite limbs are separated.

36. *The term of period* (28 ± 5) *P.*—The existence and, assuming the existence, the constants of this term are so doubtful that until further evidence of its reality is obtained, a discussion will not be useful. It is more doubtful in the coefficient of sin F than in that of cos F. It may be mentioned, however, that the computed period is nearly half of the 60 to 70 year term in the longitude, and, in the form of a perturbation of the node, there may be some connection between the two terms.

Yale University:
1914 *January* 15.

Baxendell's Observations of Variable Stars. Third Instalment. No. 6, U Cygni; No. 7, R Delphini; No. 8, S Delphini; No. 9, T Delphini. Edited by H. H. Turner and Mary A. Blagg.

No. 6, *U Cygni* (7299 *Chandler*, 201647 *Harvard*).

$20^h\ 16^m\ 30^s\ +47^\circ\ 34'{\cdot}7$ (1900·0).
15 7 26·3 (1855·0).

Identification of Comparison Stars.—In this case Baxendell kept the same designations for the comparison stars throughout, as in the first column of Table I.

TABLE I.

Comparison Stars.

Bax.	Knott 11-3-84.	A.S.V. (IV.) No.	A.S.V. (IV.) Grad.	Harv. Letter.	Harv. Mag.	Adopted Mag.
a	*a*	10	28	*e*	7·77	82
b	*b*	26	62	*h*	9·25	91
c	*c*	39	85	*l*	9·78	97
d	*d*	49	103	...	...	103
e	*e*	42	90	*m*	9·96	103
f	*f*	...	...	...	...	108
g	*g*	53	113	*q*	11·08	111
x	*z*	28	70	...	...	92
y	*h*	...	...	...	...	...

The third column gives the number and grade (according to Hagen) from Series IV. of the *Atlas Stellarum Variabilium*: the fourth column gives the Harvard Magnitudes from *H.A.*, xxxvii. p. 176.

TABLE II.

Baxendell's Differences between his Comparison Stars. (Subscript figures give numbers of Observations.)

Years.	91–82.	97–91.	103(*d*)–97.	103(*e*)–103(*d*).	108–103(*e*).	111–108.	92–91.	97–92.
1878–9	9_{17}	2_2	6_5	0_5	8_{21}	3_{21}	1_{10}	2_1
1880	10_{26}	7_1	7_2	-2_1	...	...	1_{21}	2_1
1881	11_4	6_4	6_2	$+2_1$	8_1	2_1	1_5	...
1882–3	10_4	...	...	$+3_2$	...	...	...	...
1884	12_6	6_5	6_2	0_5	7_4	3_2	4_5	3_6
1885–7	11_{11}	9_2	...	$+3_2$	...	...	2_8	**6**
Mean(*s*)	10·0	6·0	6·2	0·8	7·8	3·0	1·2	3·6
No. of Obs.	68	14	11	16	26	24	49	11
Reduced Mean (*t*)	8·0	5·0	5·2	0·7	6·6	2·6	1·1	3·1

The adopted magnitudes are a compromise between those of Harvard and those suggested by Baxendell's own comparisons when these are reduced by the formula

$$t = 0{\cdot}915s - 0{\cdot}0115s^2$$

which has been found to give, on the average, the relation between his step (*s*) and 0·1 mag. (*t*) up to intervals of about two magnitudes in amount. But he seems to have "seen" certain individual stars persistently brighter or fainter than the Harvard results would make them.

The telescope was E with power 80 throughout: *i.e.* a 6-in. equatorial 87½ in.-focus. (See List in *M.N.*, lxxiii. p. 126.)

TABLE III.

Baxendell's Observations of U Cygni.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.	Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1878				1878			
July 12	7178	103 + 3, 103 − 1, 111 − 8	104	Nov. 17	7306	82 + 1, 91 − 3	86
				19	7308	82 + 2, 91 − 3	86
Sept. 5	7233	97 − 1	96	28	7317	82 − 1	81
11	7239	82 + 9, 91 − 2	90	Dec. 4	7323	82 − 3	79
14	7242	82 + 5	87	5	7324	82 − 2	80
15	7243	82 + 7, 91 − 1	90	9	7328	82 − 1	81
Oct. 13	7271	82 + 7, 91 − 4	88	22	7341	82 − 2	80
Nov. 4	7293	82 + 2	84	30	7349	82 − 2	80
	97	82 + 5, 91 − 3	88				

TABLE III.—*Baxendell's Observations of U Cygni*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1879			
Jan. 13	7363	82 − 2	80
15	7365	82 − 1	81
19	7369	82 − 1	81
22	7372	82 + 0	82
26	7376	82 + 1	83
Apr. 6	7446	82 + 1	83
Aug. 11	7573	111 − 2	109
14	7576	111 − 1	110
17	7579	111 − 1	110
21	7583	111 − 2	109
22	7584	111 + 0	111
23	7585	111 − 2, 108 + 1	109
27	7589	103 + 2, 111 − 3, 108 + 0	107
29	7591	103 + 3, 108 − 3, 111 − 6	105
30	7592	103 + 6, 111 − 3, 108 + 0	108
31	7593	103 + 5, 111 − 5, 108 − 3	106
Sept. 2	7595	103 + 6, 111 − 4, 108 − 2	107
7	7600	103 + 7, 111 − 3, 108 − 1	108
13	7606	103 + 6, 111 − 4, 108 − 1	108
19	7612	103 + 6, 111 − 4, 108 − 1	108
21	7614	103 + 6, 111 − 4, 108 − 2	107
23	7616	103 + 7, 111 − 3, 108 − 1	108
28	7621	103 + 5, 108 − 3, 111 − 7	106
Oct. 1	7624	103 + 3, 111 − 8, 108 − 5	104
4	7627	103 + 3, 108 − 5, 111 − 8	104
Oct. 6	7629	103 + 7, 111 + 0, 108 + 2	110
14	7637	103 + 1, 111 − 8, 108 − 6	103
15	7638	103 + 1, 111 − 8, 108 − 6	103
20	7643	103 − 1, 108 − 5, 111 − 8, 103 + 3	104
25	7648	103 − 3, 111 − 11, 108 − 7	100
26	7649	103 − 2, 108 − 7, 111 − 10	101
Nov. 1	7655	97 − 1, 103 − 4, 103 − 5	98
2	7656	97 − 1	96
10	7664	97 − 1	96
13	7667	97 − 2	95
22	7676	82 + 6, 91 − 1, 92 − 1	90
24	7678	82 + 6, 91 − 2, 92 − 1	89
29	7683	82 + 6, 91 − 3, 92 − 3	88
Dec. 1	7685	82 + 6, 91 − 1, 92 − 2	89
3	7687	82 + 5, 91 − 3, 92 − 3	88
6	7690	82 + 8, 91 + 2, 92 − 2	90
21	7705	97 − 3	94
22	7706	82 + 3, 91 − 3, 92 − 4, 97 − 6	88
24	7708	82 + 2, 91 − 3, 92 − 4	87
26	7710	82 + 2, 91 − 5, 92 − 6	85
28	7712	82 + 3, 92 − 7, 91 − 6	85

TABLE III.—*Baxendell's Observations of U Cygni*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1880			
Jan. 2	7717	82 + 2, 92 − 6, 91 − 6	85
11	7726	82 + 3, 91 − 4, 92 − 4	87
20	7735	82 + 2, 92 − 7, 91 − 6	85
22	7737	82 + 1, 91 − 7, 92 − 8	84
28	7743	82 + 0, 91 − 7, 92 − 8	83
31	7746	82 + 0, 91 − 6, 92 − 7	84
Feb. 4	7750	82 + 0, 91 − 7, 92 − 8	83
5	7751	82 + 0, 91 − 7, 92 − 8	83
8	7754	82 − 1, 92 − 8, 91 − 7	83
14	7760	82 − 1, 92 − 8, 91 − 7	83
26	7772	82 − 2, 91 − 10, 92 − 10	81
Mar. 1	7776	82 − 2, 92 − 9	82
7	7782	82 − 3, 91 − 10, 92 − 10	81
17	7792	82 − 3	79
Apr. 1	7807	82 − 2, 91 − 10	80
9	7815	82 − 2, 91 − 8	82
11	7817	82 − 2	80
16	7822	82 − 2	80
17	7823	82 − 1	81
24	7830	82 + 0	82
29	7835	82 − 1, 91 − 9	82
May 1	7837	82 − 1	81
3	7839	82 − 1	81
5	7841	82 − 1	81
7	7843	82 − 1	81
9	7845	82 − 1	81
12	7848	82 − 1	81
13	7849	82 − 1	81
20	7856	82 − 1, 91 − 10, 92 − 10	81
22	7858	82 − 1	81
24	7860	82 − 1	81
June 4	7871	82 + 0	82
7	7874	82 + 0	82
17	7884	82 + 0, 91 − 9	82
26	7893	82 + 1, 91 − 8, 92 − 9	83
July 4	7901	82 + 1, 91 − 8	83
9	7906	82 + 1, 91 − 9	82
29	7926	82 + 3	85
31	7928	82 + 3, 91 − 7	84
Aug. 9	7937	82 + 3, 91 − 7, 92 − 8	84
13	7941	82 + 4, 91 − 6	86
24	7952	82 + 9, 91 + 0	91
30	7958	91 − 1, 92 − 2	90
Sept. 2	7961	91 + 1, 92 − 2	91
8	7967	91 + 0	91
18	7977	91 + 4, 92 + 4	96
27	7986	91 + 7, 92 + 2, 97 + 1	97
Oct. 2	7991	97 + 3, 103 − 1	101
10	7999	97 + 4, 103 + 0	102
23	8012	111 − 3	108
Nov. 19	8039	111 − 2	109
Dec. 4	8054	111 + 3	114
18	8068	111 + 0	111
24	8074	111 − 1	110
1881			
Jan. 6	8087	111 + 2	113
Apr. 2	8173	91 + 1, 92 + 0, 82 + 9	92
23	8194	91 + 3, 97 − 1	95

TABLE III.—*Baxendell's Observations of U Cygni*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1881			
May 4	8205	82 + 8, 91 + 0, 92 − 1	91
12	8213	91 − 1, 92 − 1	90
20	8221	82 + 7, 91 − 3, 92 − 4	88
29	8230	82 + 3, 91 − 7, 92 − 7	85
June 12	8244	82 + 1	83
18	8250	82 − 1	81
July 11	8273	82 − 1	81
20	8282	82 − 2	80
Aug. 1	8294	82 − 1	81
5	8298	82 − 2	80
24	8317	82 + 0	82
Sept. 14	8338	82 − 1	81
27	8351	82 + 2	84
Oct. 1	8355	82 + 0	82
8	8362	82 + 2	84
15	8369	82 + 2	84
20	8374	82 + 3	85
26	8380	82 + 3	85
Nov. 14	8399	91 + 3, 97 − 3	94
20	8405	91 + 4, 97 − 3	94
Dec. 19	8434	97 + 0	97
1882			
Apr. 10	8546	111 + 3	114
May 12	8578	103 + 0, 111 − 7	104
14	8580	103 + 1, 111 − 6	104
19	8585	103 + 1, 103 − 2	102
28	8594	97 + 0, 103 − 7	96
July 9	8636	82 + 3, 91 − 5	86
24	8651	82 + 3, 91 − 5	86
Sept. 4	8693	82 + 1	83
9	8698	82 + 2 91 − 6	84
15	8704	82 − 2	80
Oct. 28	8747	82 − 3	79
1883			
Apr. 8	8909	103 + 0, 103 − 3	102
July 28	9020	103 + 1, 111 − 5	105
Aug. 25	9048	97 + 6, 103 − 3	102
1884			
Feb. 10	9217	82 − 6	76
24	9231	82 − 6	76
Mar. 5	9241	82 − 5	77
16	9252	82 + 3	85
21	9257	82 − 3	79
25	9261	82 − 3	79
Apr. 7	9274	82 − 2	80
12	9279	82 − 2	80
28	9295	82 − 1	81
May 4	9301	82 + 0	82
18	9315	82 + 0	82
22	9319	82 + 4	86
June 4	9332	82 + 4, 91 − 6	86
12	9340	82 + 4	86
14	9342	82 + 6, 91 − 4	88
Aug. 7	9396	97 + 1, 103 − 5, 103 − 6	98
Sept. 5	9425	103 + 5, 108 − 2	107
11	9431	103 + 8, 108 − 3, 111 − 5	107
18	9438	108 − 1, 111 − 4	107
Oct. 5	9455	103 + 2, 108 − 1	106
12	9462	103 + 4, 108 + 0	108
Nov. 2	9483	103 − 2, 103 − 2	101
13	9494	103 − 1, 103 − 1	102
21	9502	103 − 3, 103 − 2	100
27	9508	97 − 3, 103 − 5, 92 + 3	96
Dec. 3	9514	97 − 5, 92 − 3, 91 + 3	92
4	9515	91 − 1, 92 − 3, 97 − 3	91
6	9517	91 + 1, 97 − 3, 92 − 3	92
8	9519	91 − 3, 82 + 8	89

TABLE III.—*Baxendell's Observations of U Cygni*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1884			
Dec. 14	9525	82 + 9, 91 + 0, 97 − 5, 92 − 4	90
20	9531	82 + 9, 97 − 3, 91 + 3	93
24	9535	91 + 3, 97 − 5, 92 − 4	91
1885			
Jan. 5	9547	82 + 6, 91 − 3	88
8	9550	82 + 6, 91 − 3, 92 − 5	88
14	9556	82 + 3, 91 − 5	86
Feb. 14	9587	82 + 4, 91 − 4	86
Mar. 6	9607	82 + 7, 91 − 3	88
11	9612	82 + 2, 91 − 10	82
27	9628	82 − 3	79
Apr. 1	9633	82 − 3	79
8	9640	82 − 3	79
17	9649	82 − 5	77
19	9651	82 − 5	77
26	9658	82 − 4	78
May 8	9670	82 − 4	78
11	9673	82 − 4	78
20	9682	82 − 5	77
June 1	9694	82 − 6	76
9	9702	82 − 4	78
12	9705	82 − 3	79
July 4	9727	82 − 2	80
29	9752	82 + 1	83
Aug. 14	9768	82 − 1	81
20	9774	82 + 0	82
Sept. 3	9788	82 + 0	82
8	9793	82 + 1	83
25	9810	82 + 2	84
Oct. 29	9844	82 + 9, 91 + 0, 92 + 1	92
1885			
Nov. 8	9854	91 + 4, 97 − 4, 92 + 4	95
15	9861	82 + 8, 91 − 3, 92 − 3	89
19	9865	92 + 2, 91 + 2	94
28	9874	92 + 4, 97 − 3	95
Dec. 1	9877	103 − 2	101
26	9902	103 − 2	101
1886			
Jan. 5	9912	103 − 2	101
13	9920	103 + 4, 103 + 0	105
30	9937	103 − 2	101
31	9938	103 − 1, 103 − 2	102
Feb. 14	9952	97 + 0	97
Mar. 29	9995	82 + 6, 91 − 5	87
	241+		
Apr. 6	0003	82 + 3, 91 − 7	84
26	0023	82 + 1	83
May 21	0048	82 − 2	80
29	0056	82 − 2	80
June 14	0072	82 − 3	79
26	0084	82 − 3	79
Sept. 18	0168	82 − 3	79
Oct. 23	0203	82 − 3	79
Nov. 17	0228	82 − 3	79
Dec. 4	0245	82 − 2	80
29	0270	82 + 0	82
1887			
Jan. 12	0284	82 + 0	82
28	0300	82 + 3	85
Feb. 2	0305	82 + 5, 91 − 3	88
June 7	0430	91 + 2, 92 + 1	92
10	0433	91 + 3, 92 − 1	92

The results of these observations are summarised in Table IV. in the same manner as with the stars already discussed in this series. The length of period assumed was 462 days, and this

was divided into twelve portions, as nearly equal as possible. From the mean of the observations in each portion, a smooth curve was formed, the reading from which is given for the middle point of each portion.

TABLE IV.

Comparison of the Observations with a Mean Light Curve.

Period.	Mag. from Curve. 240+.	1. 86.	2. 93°5.	3. 102°5.	4. 107.	5. 106.	6. 94°5.	7. 88.	8. 83°5.	9. 80°5.	10. 79.	11. 80.	12. 82.
I.	7000	...	...	...	...	-1_1	...	$+1_4$	$+3_4$	$+1_6$	$+2_6$	...	$+1_1$
II.	7462	...	...	$+5_2$	$+1_{13}$	$-1\cdot5_{10}$	-3_{10}	-1_7	0_8	0_4	$+2_{11}$	$+1\cdot5_8$	$+1_3$
III.	7924	$+1_7$	$+3_5$	$+7_1$	$+4_4$	$+6_1$	...	$+4_1$	$+6\cdot5_5$	$+1_2$	$+1\cdot5_4$	$+1\cdot5_2$	$+2_6$
IV.	8386	$+9_2$	$+5\cdot5_1$	.	...	$+6\cdot5_1$	$+3_4$	$-1\cdot5_2$	$+1_1$	$+1_2$	0_1	...	...
V.	8848	...	$+7_1$	...	...	-1_1	$+3\cdot5_1$	...	...	...	-3_2	0_4	-1_4
VI.	9310	-1_5	...	-2_1	0_5	-3_2	-3_8	-2_3	$+3_3$	-2_5	-2_5	-1_3	-1_2
VII.	9772	-3_3	-5_2	-7_5	-5_3	-6_3	$-5\cdot5_1$	-5_2	-4_3	-2_1	...	$-0\cdot5_1$	-3_2
	241+												
VIII.	0234	-6_2	-10_3	...	...	...	-7_2	...	...	...	...	...	...
	$p-f$	−11°5	−16°5	−13°5	−3°5	+12°5	+18	+11	+7°5	+4°5	+0°5	−3	−6

The periods being so few in number, and so unequally observed, a mean correction to epoch formed from several consecutive periods would have little value. The maximum in the mean curve is obviously near the middle of portion 10, so corrections to epoch have been obtained from the best observed periods individually, in such a way as to have the epoch of max. about in the middle of the set of "portions" taken. Four periods have been taken, as follows:—

		Correction to Period.
Period	II. (II. 4 to III. 4); correction to epoch = − 10°9 days	
	III. (III. 3 to IV. 5); ,, ,, = − 7°1 ,,	+3°8
	VI. (VI. 4 to VII. 4); ,, ,, = + 3°1 ,,	+3°4(3)
	VII. (VII. 4 to VIII. 6); ,, ,, = + 5°5 ,,	+2°4

The last is of doubtful value, the observations being badly distributed. The corrections have been weighted by "$p-f$" and by the number of observations.

Chandler's formula is 2404596 + 461·3E, which does not give any variation in length of period. Some such variation as is found in most of these cases would, however, suit the above figures very well. The following Table gives the comparison between Baxendell and Chandler, the epoch of max. in the mean curve being taken at 363 days after (or 99 days before) the zero point, though it might easily be read differently by at least 6 days either way.

Period.	E.	Chandler's Epoch.	Baxendell's Epoch.	Bax. – Ch. d.
II.	+ 7	2407825·1	7462+363 – 10·9=7814·1	– 11·0
III.	+ 8	8286·4	7924+363 – 7·1=8279·9	– 6·5
VI.	+11	9670·3	9310+363+ 3·1=9676·1	+ 5·8
VII.	+12	2410131·6	9772+363+ 5·5=0140·5	+ 8·9

Chandler's period is 0·7 days shorter than the period assumed in the above discussion, which would account for a progressive correction of – 3·5 days in 5 periods. The progressive correction is, however, quite the other way about, and the average length of period during these observations appears to be $462+\frac{16\cdot4}{5}$, or about 465·3 days. While these results strongly suggest an epoch variability of the usual kind, the data are insufficient for ascertaining the value of either C or A, even approximately, there being nothing to show how much further the difference would extend. Moreover, the figures are fairly compatible with the theory of an unvarying period of about 465·3 days.

A correction to epoch has been obtained also for Period V (V 2 to VI 6), amounting to – 0·9 days, and giving Bax.–Ch. = + 1·1 days. Though less well observed, and less well distributed, this fits in fairly well with the other corrections. Analysis of the mean curve gives the formula

$$1\cdot35\cos(\theta-94^\circ)+0\cdot32\cos 2(\theta-93^\circ)+0\cdot10\cos 3(\theta-97^\circ).$$

The zero of the expression is the middle of the first portion of the mean curve, 19 days after the starting point of a period and 118 days (= 92°) after the assumed place of maximum. The two best observed periods (II 4 to III 4 + VI 4 to VIII 4) have been analysed separately, with the following results, which are here compared with the formula for the mean curve. The coefficients have been divided by the range and magnitude, and multiplied by 6.

Mean curve $2\cdot79\cos(\theta-94^\circ)+0\cdot66\cos 2(\theta-93^\circ)+0\cdot21\cos 3(\theta-97^\circ)$
Period II. $2\cdot71\cos(\theta-85^\circ)+0\cdot89\cos 2(\theta-82^\circ)+0\cdot29\cos 3(\theta-76^\circ)$
Period VI. $2\cdot53\cos(\theta-101^\circ)+0\cdot62\cos 2(\theta-98^\circ)+0\cdot58\cos 3(\theta-94^\circ)$

The differences in these formulæ are in pretty good agreement with the corrections to epoch, so far as the first two harmonics are concerned.

No. 7, R Delphini (called by Baxendell, S Aquilæ)
(7261 Chandler, 201008 Harvard).

$20^h\ 10^m\ 5^s\ +8^\circ\ 47'\cdot1$ (1900·0).
$20\ \ 7\ \ 55\ \ +8\ \ 39\ '\cdot1$ (1855·0).

TABLE V.

Comparison Stars.

Bax.	A.S.V. (II.) No.	A.S.V. (II.) Grad.	Harv. Mag.	Adopted Mag.
A	..	...	...	...
a	4	11	8·81	88
b	7	21	9·18	93
v	10	28	9·42	95
c	14	45	9·98	100
x	15	50	10·13	101
w	19	61	10·41	103
t	23	83	10·93	109
q	24	87	11·03	110
y	32	113	11·57	116

"A" is used only once, and there are no sufficient data for its identification. The magnitudes adopted are those of Harvard, with some slight alterations to suit Baxendell's comparisons a little better. The Harvard magnitudes have been interpolated from the table given in *H.A.*, xxxvii. p. 200, as the lists on pp. 175 and 196 do not contain enough of the Baxendell stars.

TABLE VI.

Baxendell's Differences between his Comparison Stars.

Year.	93–88.	95–93.	100–95.	103–95.	103–101.	116–110.
1863	11_1	2_2	...	3_2	0_1	9_1
1864	11_4	6_1	...	...	...	19_1
1865–6	6_3	4_1	...	...	...	14_3
1868	6_4	...	...	...	...	...
1878–9	12_1	3_1	5_8	4_{11}	2_3	...
Mean	8_{13}	3_5	5_8	4_{13}	1_4	14_5
Reduced Mean	7	2	4	3	1	11

The reduced means are obtained by the application of the usual formula to each comparison. They do not agree very well with the Harvard magnitudes in this case, and they are more than usually discordant in different years.*

The telescope used was A 68 until the end of 1868 and E 80 afterwards (see list on p. 126 of *M.N.*, lxxiii., or 451 of *M.N.*, lxxiv.).

* 103–95 ranges from 0·2 to 1·0 mag.

TABLE VII.

Baxendell's Observations of R Delphini.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1863			
Nov. 10	1820	116 − 2	114
11	1821	110 + 4, 116 − 3	114
28	1838	100 + 0, 116 − 12	102
29	1839	100 − 1	99
Dec. 17	1857	93 + 0, 95 − 2	93
22	1862	93 − 3, 95 − 5	90
27	1867	88 + 5, 93 − 4	91
1864			
May 16	2008	Not seen < 120	
July 4	2057	,, < 130	
13	2066	,, < 132	
29	2082	,, < 132	
Aug. 21	2105	? Glimpsed < 130	
23	2107	? Right star 132	
31	2115	Glimpsed 132	
Sept. 5	2120	116 + 6	122
24	2139	116 + 3	119
Oct. 5	2150	110 + 3, 116 − 11	109
8	2153	110 + 0	110
24	2169	88 + 11, 93 + 1 95 − 4	95
31	2176	88 + 7, 93 − 1	94
Nov. 5	2181	88 + 5, 93 − 2	92
8	2184	88 + 7, 93 − 2	93
1865			
May 18	2375	Not seen < 125	
June 9	2397	,, < 115	
17	2405	Glimpsed 130	
20	2408	Glimpsed 130	
July 1	2419	116 + 0	116
14	2432	110 + 1, 116 − 9	108
21	2439	100 + 0	100
Aug. 3	2452	95 − 3	92
12	2461	93 − 3, 95 − 6	90
25	2474	88 − 1, 93 − 5	88
26	2475	88 − 1	87
31	2480	88 − 2	86
Sept. 7	2487	88 − 1	87
23	2503	93 + 0	93
27	2507	93 + 3, 103 − 2	98
Oct. 2	2512	93 + 3, 103 − 2	98
10	2520	103 + 3, 100 − 1	103
Nov. 1	2542	110 + 2	112
7	2548	110 + 2	112
15	2556	110 + 9, 116 − 3	116
1866			
May 20	2742	88 − 2	86
21	2743	88 − 1	87
June 8	2761	88 − 2	86
25	2778	88 + 3	91
July 11	2794	88 + 8, 95 − 2	94
16	2799	95 − 1	94
31	2814	109 + 1, 95 + 9	107
Aug. 31	2845	116 + 0	116
Sept. 8	2853	116 − 1	115
Oct. 6	2881	116 + 6	122
Nov. 9	2915	Not visible < 132	
1867			
July 27	3175	Glimpsed 131	
1868			
June 4	3488	Not seen < 125	
16	3500	,, < 125	
July 11	3525	Glimpsed 132	
Sept. 5	3581	93 + 2, 88 + 7	95
6	3582	88 + 7, 93 + 0	94
Oct. 17	3623	88 − 2	86
19	3625	88 − 1	87
27	3633	88 − 1	87
Nov. 8	3645	88 + 4, 93 + 0	92
15	3652	88 + 5, 93 + 0	93
1878			
Aug. 8	7205	Invisible < 125	
Nov. 2	7291	116 − 1	115
8	7297	95 + 2, 100 − 2	98

TABLE VII.—*Baxendell's Observations of R Delphini*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1878			
Nov. 28	7317	88 − 2	86
Dec. 5	7324	88 − 6	82
7	7326	88 − 5, (?) + 8	83
30	7349	88 + 6, 93 − 4	92
1879			
Aug. 17	7579	100 + 2, 101 + 1	102
31	7593	88 + 2, 95 − 5, 100 − 8, 93 − 3	90
Sept. 2	7595	88 + 1, 95 − 4	90
1879			
Sept. 7	7600	88 − 1, 95 − 3	90
9	7602	88 − 1, 95 − 5	88
13	7606	88 + 0, 95 − 5	89
19	7612	88 + 0, 95 − 5	89
23	7616	88 − 1, 95 − 5	88
25	7618	88 − 1, 95 − 6	88
Oct. 5	7628	88 + 2, 95 − 5	90
15	7638	88 + 2, 95 − 5	90
Nov. 5	7659	95 + 4, 100 − 3	98

The observations have been tabulated in the same manner as with the other variables, but there are difficulties in dealing with them, owing to the fact that Baxendell was unable to observe the star near its minimum, as it was then too faint to be visible in his instruments. The tabulated observations show that there is an apparent shift forward in the phase of one "portion,"—or about twenty-four days—as between periods XX. and XXI. and periods II. to V. The mean curve has therefore been formed by shifting the observations of the two latest periods one portion back. The observations in period I. are so discordant with the rest that they have not been used in forming the curve. The single observation in portion (2) was taken near the end of that portion, and has been placed in its right position in the curve, instead of in the centre of the portion. The results of comparing the observations with the adopted curve are shown in Table VIII.

TABLE VIII.

Comparison of the Observations with a Mean Light Curve.

	240+.	1. 12.	2. 35.	3. 59.	4. 83.	5. 106.	6. 130.	7. 153.	8. 177.	9. 201.	10. 224.	11. 248.	12. 272.
I.	1700	...	...	...	...	...	-15_4	-20_2	-16_1	...	...	...	...
II.	1984	...	...	...	...	...	$+1_1$	$+5_1$	$+2_3$	$+3_3$	...	...	...
III.	2268	...	...	...	...	...	...	0_2	-4_2	-2_4	-1_2	$+4_3$	-1_2
IV.	2552	$+1_1$	...	...	...	...	...	...	...	-4_3	$+1_1$	-1_2	$+4_1$
V.	2836	-1_2	0_1	...	...	...	...	...	...	...	...	...	...
VI.	3120	...	...	...	...	...	...	...	...	...	...	...	...
VII.	3404	...	...	...	...	...	...	...	-5_2	...	-2_3	-2_2	...
XX.	7096	...	...	...	...	...	...	...	...	$+9_1$	-6_3	-6_1	...
XXI.	7380	...	...	...	...	..	...	...	...	$+13_1$	0_7	-7_4	-14_1
Mean II+(2)		+1	...	...	...	...	+1	+2	−1	−1	0	+2	+1
Mean V+(2)		−1	0	...	...	...	...	...	−5	...	−2	−2	..
Adopted Curve		116	121	...	...	...	122	115·5	99	88	90	96	109·5
$p-f$		−12	...	...	...	...	...	+23	+28	+9	−8	−20	−20

The maximum is well marked in the curve, and appears to be at about 207 days after the zero point.

As a fourth part of the curve is missing, it is not possible to analyse it harmonically; but the periods have been compared with one another and with Chandler's formula, from the data shown in Table VIII. For the mean of periods II. to IV. we get a correction to epoch of $-1{\cdot}2$ days; for the mean of periods V. and VII. (which are very scantily observed) a correction of $-2{\cdot}4$ days; for period XX., one of $+20{\cdot}6$ days; and for period XXI., one of $+25{\cdot}7$ days.

Chandler's formula is

$$2402490 + 284{\cdot}4E + 26 \sin (9^\circ E + 200^\circ).$$

The result of the comparison is as follows:—

Baxendell. d.	Mean E	Chandler. d.	Bax.–Ch. d.
$2402475 - 1{\cdot}2 = 2473{\cdot}8$	0	$2402490 - 8{\cdot}9 = 2481{\cdot}1$	$-7{\cdot}3$
$3327 - 2{\cdot}4 = 3324{\cdot}6$	3	$3343{\cdot}2 - 19{\cdot}0 = 3324{\cdot}2$	$+0{\cdot}4$
$7303 + 20{\cdot}6 = 7323{\cdot}6$	17	$7324{\cdot}8 - 3{\cdot}2 = 7321{\cdot}6$	$+2{\cdot}0$
$7587 + 25{\cdot}7 = 7612{\cdot}7$	18	$7609{\cdot}2 + 0{\cdot}9 = 7610{\cdot}1$	$+2{\cdot}6$

The agreement is remarkably close, considering the fragmentary nature of Baxendell's observations. The assumed period was 284 days, so that the average length of period between II. and XXI. would be for Baxendell $284 + \frac{26{\cdot}9}{18} = 285{\cdot}5$ days, and that for Chandler $284{\cdot}4 + \frac{9{\cdot}8}{18} = 284{\cdot}9$ days. But Baxendell's data are too incomplete to allow any important deductions to be drawn from the comparison.

No. 8, S Delphini (called erroneously R Delphini by Baxendell, from 1863 Dec. 22: 7431 Chandler, 203816 Harvard).

$20^h\ 38^m\ 28^s\ +16^\circ\ 43'{\cdot}7$ (1900·0).
$20\ \ 36\ \ 24\ \ +16\ \ 34\ {\cdot}2$ (1855·0).

1. *The Comparison Stars.*—Baxendell gives three rough sketches and a chart for the comparison stars, but the same designations are used throughout. They have been identified as in Table IX.

TABLE IX.

Comparison Stars.

Bax.	A.S.V. No.	A.S.V. Grad.	H.A. Mag.	Adopted Mag.
a	5	7	8·66	87
b	4	7	8·58	88
c	8	25	9·55	95
d	17	43	10·09	100
e	14	37	9·90	99
f	22	53	...	104
g	26	55	...	108
x	23	54	10·70	107
y	27	60	10·96	112

By A.S.V. is meant Hagen's *Atlas Stellarum Variabilium*, and by H.A., *Harvard Annals*, lvii. p. 264. The *adopted mag.* is that which has been used throughout to designate the stars in the observations, in Table XI.; the decimal point having been omitted.

Baxendell's differences between his comparison stars are shown in Table X., as was done in the cases of the stars already dealt with in this series.

TABLE X.

Baxendell's Differences between his Comparison Stars.

Year.	88–87.	95–87.	100–95.	100–99.	104–99.	108–104.	107–104.	112–107.
1860–1	4_5	11_3	5_3	2_1	11_1	1_2	3_{25}	...
1862–3	1_{12}	12_{11}	5_6	1_2	...	...	3_{27}	8_1
1864–5	...	11_{19}	4_4	...	...	...	2_6	5_{13}
1866–7	...	8_{22}	...	...	7_3	...	2_4	6_{11}
1868–9	...	7_{13}	...	...	3_1	...	...	12_2
1877–9	...	9_{43}	4_{19}	1_{10}	6_4	4_{21}	3_4	...
1880–1	...	9_{26}	3_2	0_4	4_3	5_{26}	...	...
1882–3	...	8_7	...	...	...	5_4	3_4	...
1884–6	...	9_{19}	7_1	...	7_2	5_{16}	...	...
Mean	1·9	9·1	4·1	0·8	6·1	4·6	2·8	6·0
No. of Obs.	17	163	35	17	14	69	70	27
Reduced Means.	1·6	7·1	3·6	0·8	4·6	3·9	2·6	4·9

The reduced means are found by applying to each individual comparison the scale of conversion of Baxendell's "steps" into tenths of a magnitude which has been used throughout the series. The magnitudes adopted are a compromise between those suggested by these means and those given in *H.A.*

The telescopes used were A 68 until the end of 1869, and E 80 from 1877; with the following exceptions:—C 39 was used on 1861 Aug. 6; 1862 April 29, July 4, 19, 24, and Sept. 26; 1863 June 12, 17, July 17, 18, 20, and Sept. 4 and 19: R (13) 81 was used on 1861 May 4, 9, and June 13 and 14: 22 (*a*) 39 was used on 1875 Dec. 5: and E with power 110 on 1877 Dec. 31.

TABLE XI.

Baxendell's Observations of S Delphini.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.	Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1860				1861			
Nov. 22	0737	87 + 9, 95 + 3, 100 − 2	97	Jan. 4	0780	100 + 7, 87 + 19, 104 + 0, 107 − 2	106
Dec. 18	0763	87 + 20, 100 + 6, 107 − 2	106	6	0782	104 + 1, 107 − 2, 108 + 0	106
20	0765	100 + 7, 107 − 2, 104 − 1	105	May 4	0900	104 + 3, 107 + 0	107
				9	0905	104 + 0, 107 − 3	104

TABLE XI.—*Baxendell's Observations of S Delphini*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1861			
May 17	0913	107 − 3, 104 − 1, 100 + 0	102
27	0923	104 − 3, 100 − 1, 107 − 5	101
June 1	0928	95 − 1, 100 − 3, 107 − 7	97
13	0940	87 + 3	90
14	0941	87 + 3	90
16	0943	87 + 3	90
19	0946	87 + 3, 88 + 0	89
25	0952	87 + 3, 88 + 0	89
30	0957	87 + 2, 88 − 2	88
July 23	0980	87 + 0	87
26	0983	87 + 0	87
30	0987	87 + 0	87
Aug. 5	0993	87 + 1	88
6	0994	87 + 2	89
21	1009	88 + 4, 95 − 2, 87 + 8	93
28	1016	88 + 6, 95 − 1, 87 + 9	95
30	1018	95 − 1	94
Sept. 12	1031	95 + 4, 104 − 3	100
20	1039	104 − 2, 107 − 3	103
23	1042	100 + 0, 104 − 1, 107 − 2, 95 + 8, 87 + 18	103
Oct. 5	1054	104 + 2, 107 − 1	106
9	1058	104 + 2, 107 − 1	106
11	1060	104 + 1, 107 − 1	106
13	1062	104 + 2, 107 − 1	106
14	1063	104 + 2, 107 − 1	106
16	1065	104 + 3, 107 − 1	106
19	1068	104 + 2, 107 − 1	106
22	1071	104 + 3, 107 − 1	106
27	1076	104 + 3, 107 − 1	106
28	1077	104 + 3, 107 + 0	107
Nov. 2	1082	104 + 3, 107 + 0	107
Nov. 11	1091	104 + 4, 107 + 1	108
15	1095	104 + 4, 107 + 0	108
17	1097	104 + 4, 107 + 1	108
27	1107	104 + 4, 107 + 0	108
30	1110	104 + 4, 107 + 0	108
Dec. 3	1113	104 + 4, 107 + 0	108
21	1131	107 + 1	108
25	1135	107 + 2	109
1862			
Apr. 29	1260	87 + 2	89
June 28	1320	104 + 1, 107 − 2	105
July 4	1326	> 104 Barely glimpsed.	
19	1341	104 + 0	104
24	1346	> 107	
26	1348	107 + 2, 104 + 2	108
27	1349	107 + 2, 104 + 3	108
28	1350	104 + 3, 107 + 0	107
Aug. 14	1367	104 + 2, 107 + 0	106
22	1375	104 + 3, 107 + 0	107
25	1378	104 + 2, 107 + 0	106
Sept. 1	1385	104 + 2, 107 + 0	106
10	1394	104 + 2, 107 + 0	106
15	1399	104 + 2, 107 − 2	106
17	1401	104 + 2, 107 − 2	106
18	1402	104 + 3, 107 − 2	106
20	1404	104 + 3, 107 + 0	107
26	1410	104 + ?, 107 − ?	
Oct. 4	1418	107 + 0	107
8	1422	107 + 0	107
18	1432	104 + 2, 107 + 0	106
24	1438	104 + 2, 107 + 0	106
29	1443	104 + 2, 107 − 1	106
Nov. 2	1447	104 + 2, 107 − 1	106
10	1455	104 + 1, 107 − 2	105
16	1461	104 + 1, 107 − 1	106
23	1468	104 + 2, 107 − 1	106
24	1469	104 + 1, 107 − 2	105
Dec. 26	1501	104 − 7, ? − 1	97

TABLE XI.—*Baxendell's Observations of S Delphini*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1863			
Jan. 2	1508	87 + 8 95 − 1, 100 − 5	95
23	1529	87 + 4, 95 − 9, 88 + 1	89
May 23	1649	107 + 5	112
June 3	1660	< 110	
12	1669	< 100	
17	1674	< 115	
20	1677	107 + 6	113
23	1680	107 + 3	110
25	1682	107 + 2	109
30	1687	107 + 3	110
July 3	1690	107 + 0	107
5	1692	107 + 0	107
17	1704	< 105	
18	1705	107 − 1	106
20	1707	104 + 0, 107 − 2	104
28	1715	107 − 3, 104 − 1	104
29	1716	104 + 0, 107 − 2	104
31	1718	104 − 1, 107 − 3	104
Aug. 4	1722	100 + 4, 104 − 1, 107 − 3	104
13	1731	100 + 3, 104 − 2, 107 − 4	103
15	1733	100 + 2, 104 − 3	102
20	1738	104 − 3, 100 + 0	100
24	1742	100 + 2, 104 − 3	102
29	1747	100 − 1	99
Sept. 3	1752	100 + 0, 104 − 3	100
4	1753	100 + 0	100
7	1756	100 + 0, 104 − 3	100
10	1759	100 + 0	100
19	1768	100 + 0	100
22	1771	95 + 2, 100 − 3	97
Oct. 9	1788	87 + 8, 95 − 3	94
11	1790	87 + 7, 95 − 2	94
13	1792	87 + 3, 95 − 3	91
24	1803	87 + 0	87
Oct. 28	1807	87 + 1	88
29	1808	87 + 1	88
30	1809	87 + 1	88
Nov. 1	1811	87 + 1	88
2	1812	87 + 0	87
6	1816	87 + 0	87
9	1819	87 + 0	87
10	1820	87 + 0	87
11	1821	87 + 0	87
19	1829	87 + 3	90
28	1838	87 + 3, 88 + 0	89
29	1839	87 + 3	90
Dec. 5	1845	87 + 6, 95 − 3	92
17	1857	87 + 6, 95 − 3	92
22	1862	87 + 7, 95 − 1	94
27	1867	100 + 2, 104 − 3	102
1864			
Jan. 1	1872	100 + 3, 104 − 2	102
May 16	2008	104 + 1, 107 − 1	106
26	2018	104 + 1, 107 − 1	106
June 6	2029	104 − 3	101
16	2039	104 − 6, 100 − 3, 95 + 0	97
July 4	2057	95 + 0, 100 − 3	96
8	2061	95 + 0, 100 − 4	96
13	2066	87 + 7, 95 − 3	93
19	2072	87 + 6, 95 − 2	93
26	2079	87 + 6, 95 − 3	92
29	2082	87 + 5, 95 − 3	92
Aug. 21	2105	87 + 1	88
23	2107	87 + 1	88
26	2110	87 + 1	88
31	2115	87 + 1	88
Sept. 5	2120	87 + 1	88
9	2124	87 + 2	89
11	2126	87 + 3	90
14	2129	87 + 3, 95 − 6	90
19	2134	87 + 6, 95 − 4	92

TABLE XI.—*Baxendell's Observations of S Delphini*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1864.			
Sept. 24	2139	87 + 6, 95 − 4	92
26	2141	87 + 6, 95 − 3	92
Oct. 3	2148	87 + 7, 95 − 2	94
5	2150	87 + 7, 95 − 2	94
8	2153	87 + 9, 95 − 1	95
9	2154	87 + 9, 95 − 1	95
13	2158	87 + 10, 100 − 3, 95 + 0	96
31	2176	104 + 0, 107 − 2	104
Nov. 5	2181	104 + 1, 107 − 1	106
23	2199	107 + 2, 112 − 2	110
1865			
May 20	2377	87 + 2	89
June 8	2396	87 + 3	90
9	2397	87 + 3, 95 − 5	90
11	2399	87 + 3, 95 − 6	90
17	2405	87 + 3, 95 − 5	90
21	2409	87 + 3, 95 − 5	90
28	2416	87 + 2, 95 − 5	90
July 1	2419	87 + 3, 95 − 4	90
14	2432	87 + 8, 95 − 1	94
Aug. 3	2452	104 − 1, 107 − 2	104
12	2461	104 + 0, 107 − 1	105
25	2474	107 + 1	108
31	2480	107 + 4, 112 − 2	110
Sept. 7	2487	107 + 4, 112 − 4	110
23	2503	112 + 2	114
24	2504	112 + 3, 107 + 4	113
27	2507	107 + 5, 112 + 2	113
Oct. 8	2518	112 + 3	115
10	2520	112 + 2, 107 + 6	114
19	2529	112 + 3, 107 + 6	114
Nov. 5	2546	107 + 4, 112 + 0	112
7	2548	112 + 0, 107 + 4	112
12	2553	107 + 5, 112 + 0	112
15	2556	107 + 4, 112 + 0	112
21	2562	107 + 3, 112 − 2	110
26	2567	107 + 3	110
Dec. 22	2593	104 + 0, 107 − 3	104
31	2602	100 + 3, 104 − 6	100
1866			
May 17	2739	104 + 0, 107 − 2	104
20	2742	104 + 0, 107 − 2	104
24	2746	104 + 0, 107 − 2	104
June 8	2761	107 + 1, 104 + 3	108
25	2778	107 + 6, 112 + 3	114
July 11	2794	107 + 7, 112 + 3	114
16	2799	112 + 5	117
22	2805	107 + 8, 112 + 4	116
31	2814	107 + 9, 112 + 4	116
Aug. 27	2841	107 + 2, 112 − 3	109
31	2845	107 + 0, 112 − 4	108
Sept. 8	2853	104 − 1	103
15	2860	99 + 4, 104 − 2	102
24	2869	95 + 0, 104 − 9	95
Oct. 6	2881	87 + 5, 95 − 3	92
9	2884	87 + 3, 95 − 3	91
14	2889	87 + 3	90
16	2891	87 + 3, 95 − 2	92
19	2894	87 + 4, 95 − 3	92
22	2897	87 + 5, 95 − 3	92
28	2903	87 + 5, 95 − 2	92
Nov. 6	2912	87 + 4, 95 − 2	92
9	2915	87 + 6, 95 − 1	94
11	2917	87 + 6, 95 − 1	94
19	2925	87 + 7, 95 − 1	94
25	2931	87 + 7, 95 − 1	94
30	2936	87 + 7, 95 + 0	94
Dec. 16	2952	87 + 9, 95 + 0	96
30	2966	95 + 6, 104 − 3	101
1867			
Jan. 11	2978	99 + 3, 104 − 3	102
May 21	3108	107 + 6, 112 + 0	112
June 5	3123	104 + 0	104
29	3147	99 + 0, 104 − 5	99
July 24	3172	87 + 3	90

TABLE XI.—*Baxendell's Observations of S Delphini*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1867			
July 27	3175	87 + 3, 95 − 3	91
Aug. 1	3180	87 + 3, 95 − 3	91
6	3185	87 + 2, 95 − 3	90
23	3202	87 + 3, 95 − 3	91
Sept. 2	3212	87 + 6, 95 − 1	94
16	3226	87 + 6, 95 + 0	94
24	3234	87 + 6, 95 − 1	94
Oct. 17	3257	87 + 5, 95 − 1	93
Nov. 1	3272	87 + 6, 95 − 1	94
1868			
May 20	3473	87 + 3, 95 − 3	91
25	3478	87 + 3, 95 − 3	91
June 2	3486	87 + 3, 95 − 3	91
4	3488	87 + 3, 95 − 3	91
12	3496	87 + 3, 95 − 3	91
15	3499	87 + 4, 95 − 3	92
19	3503	87 + 4, 95 − 2	92
26	3510	87 + 4, 95 − 3	92
29	3513	87 + 5, 95 − 2	92
July 10	3524	87 + 4, 95 − 2	92
23	3537	87 + 3, 95 − 3	91
25	3539	87 + 2, 95 − 3	90
29	3543	87 + 3, 95 − 3	91
Sept. 4	3580	95 + 2	97
21	3597	104 − 1	103
Oct. 17	3623	107 + 7	114
Nov. 8	3645	107 + 10, 112 + 0	114
15	3652	107 + 11, 112 + 2	116
1869			
June 8	3857	104 − 1	103
Oct. 26	3997	99 + 0, 104 − 3	100
Nov. 28	4030	87 + 3, 95 − 2	92
1875			
Dec. 5	6228	95 + 0	95
1877			
Dec. 14	6968	107 + 1	108
23	6977	107 − 2	105
31	6985	107 − 3, 108 − 1	106
1878			
June 23	7159	104 − 1	103
28	7164	100 + 3, 104 − 3	102
July 7	7173	104 + 2, 108 − 1, 107 + 0	107
8	7174	108 − 2	106
12	7178	104 + 1, 108 − 2	106
20	7186	104 + 2, 108 − 1	106
27	7193	104 + 3, 108 + 0	108
Aug. 4	7201	108 + 0	108
8	7205	108 − 1	107
14	7211	104 + 2, 108 − 1	106
18	7215	104 + 3, 108 + 0	108
25	7222	108 + 0	108
28	7225	104 + 5, 108 − 1	108
Sept. 2	7230	108 + 0	108
15	7243	104 + 3, 108 − 3	106
24	7252	104 + 3, 108 − 2	106
Oct. 13	7271	104 + 1, 108 − 2	106
17	7275	104 − 1, 108 − 2	104
26	7284	104 + 1, 108 − 3	105
Nov. 1	7290	104 + 2, 108 − 3	106
8	7297	100 + 5, 104 − 3, 99 + 3	103
Dec. 5	7324	95 + 3, 100 − 3	98
7	7326	95 + 2, 100 − 3, 99 − 2	97
8	7327	87 + 5, 95 − 1	93
23	7342	95 + 4, 100 − 3	98
30	7349	87 + 1,	88
1879			
Jan. 13	7363	87 + 3, 95 − 1	92
19	7369	87 + 3, 95 − 1	92
22	7372	87 + 3, 95 − 1	92
Aug. 1	7563	104 + 1, 108 − 2	106
8	7570	104 + 1, 108 − 2, 107 − 1	106
11	7573	104 + 1, 108 − 2, 107 − 2	105

TABLE XI.—*Baxendell's Observations of S Delphini*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1879			
Aug. 14	7576	104 + 0, 108 − 3, 107 − 2	105
17	7579	104 + 0, 108 − 3	104
21	7583	99 + 4, 104 − 2, 108 − 3	103
29	7591	87 + 5, 95 − 2	92
31	7593	87 + 6, 95 − 1	94
Sept. 2	7595	87 + 5, 95 − 1, 100 − 2	95
7	7600	87 + 4, 95 − 3	92
9	7602	87 + 2, 95 − 6	89
12	7605	87 + 2, 95 − 6	89
13	7606	87 + 2, 95 − 5	90
19	7612	87 + 3, 95 − 3	91
21	7614	87 + 3, 95 − 3	91
23	7616	87 + 3, 95 − 3	91
26	7619	87 + 3, 95 − 3	91
28	7621	87 + 3, 95 − 4	90
29	7622	87 + 3, 95 − 3	91
Oct. 2	7625	87 + 3, 95 − 3	91
4	7627	87 + 3, 95 − 3	91
5	7628	87 + 3, 95 − 3	91
6	7629	87 + 3, 95 − 3	91
14	7637	87 + 4, 95 − 2	92
15	7638	87 + 4, 95 − 2	92
17	7640	87 + 5, 95 − 2	92
20	7643	87 + 5, 95 − 3	92
26	7649	87 + 5, 95 − 3	92
28	7651	87 + 5, 95 − 3	92
Nov. 1	7655	87 + 6, 95 − 3	92
2	7656	87 + 6, 95 − 2	93
5	7659	87 + 5, 95 − 3	92
10	7664	87 + 7, 95 − 2	94
12	7666	87 + 6, 95 − 3	92
13	7667	87 + 6, 95 − 3	92
22	7676	87 + 7, 95 − 1	94
23	7677	87 + 8, 95 + 0	95
24	7678	87 + 9, 95 + 1	96
1879			
Nov. 25	7679	87 + 10, 95 + 2, 99 − 1	97
27	7681	87 + 9, 95 + 1	96
29	7683	87 + 10, 95 + 3	98
30	7684	87 + 10, 95 + 2	97
Dec. 3	7687	87 + 11, 95 + 4, 104 − 6, 99 + 2	99
22	7706	87 + 10, 95 + 1	96
23	7707	95 + 4, 104 − 2	100
24	7708	104 − 2, 108 − 3	104
26	7710	95 + 5, 104 − 3, 108 − 4	102
1880			
Jan. 2	7717	104 − 1, 108 − 3	104
Apr. 17	7823	107 + 0, 104 + 4, 108 + 0	108
24	7830	108 − 1, 107 + 0	107
29	7835	108 − 1	107
May 5	7841	104 + 4, 108 − 1	108
12	7848	104 + 4, 108 − 1	108
28	7864	95 + 0	95
June 4	7871	87 + 8, 95 − 1	94
7	7874	95 + 0	95
17	7884	87 + 3, 95 − 3	91
26	7893	87 + 5, 95 − 4	92
July 4	7901	87 + 1, 95 − 6	88
9	7906	87 + 1, 95 − 7	88
20	7917	87 + 0, 95 − 7	88
30	7927	87 + 1, 95 − 6	88
Aug. 6	7934	87 + 2, 95 − 5	90
9	7937	87 + 3, 95 − 3	91
11	7939	87 + 1, 95 − 6	88
14	7942	87 + 3, 95 − 4	90
17	7945	87 + 3, 95 − 4	90
22	7950	87 + 5, 95 − 3	92
24	7952	87 + 8, 95 − 2	94
30	7958	87 + 7, 95 − 1	94
Sept. 2	7961	87 + 7, 95 − 2	94

TABLE XI.—*Baxendell's Observations of S Delphini*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1880			
Sept. 3	7962	87 + 6, 95 − 2	93
7	7966	87 + 7, 95 + 0	94
16	7975	87 + 9, 95 + 0	96
18	7977	87 + 8, 95 − 1	94
26	7985	100 + 0, 104 − 3, 108 − 6	101
Oct. 3	7992	104 − 1, 108 − 5	103
9	7998	104 + 3, 108 − 3	106
20	8009	104 + 1, 108 − 3	105
23	8012	104 + 2, 108 − 3	106
29	8018	104 + 2, 108 − 2	106
Nov. 19	8039	104 + 0, 108 − 4	104
20	8040	104 + 3, 108 − 3	106
26	8046	104 + 3, 108 − 2	106
Dec. 4	8054	104 + 2, 108 − 3	106
24	8074	104 + 3, 108 − 2	106
1881			
Jan. 12	8093	108 − 2	106
May 7	8208	87 + 2	89
12	8213	87 + 1	88
16	8217	87 + 1	88
20	8221	87 + 2, 95 − 5	90
21	8222	87 + 3, 95 − 5	90
22	8223	87 + 3, 95 − 3	91
29	8230	87 + 3, 95 − 3	91
June 8	8240	87 + 6, 95 − 1	94
12	8244	87 + 8, 95 − 1	94
18	8250	95 + 2, 104 − 3, 108 − 3	101
July 8	8270	104 − 2, 108 − 5	102
14	8276	104 − 1	103
19	8281	104 + 2, 108 − 3	106
27	8289	104 + 2, 108 − 2	106
Aug. 1	8294	104 + 3, 108 − 2	106
5	8298	104 + 2, 108 − 3	106
24	8317	104 + 3, 108 − 2	106
31	8324	104 + 5, 108 + 0	108
Sept. 1	8325	104 + 5, 108 + 0	108

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1881			
Sept. 14	8338	108 + 1	109
19	8343	108 + 0	108
25	8349	108 − 1	107
Oct. 1	8355	108 + 1	109
15	8369	108 − 1	107
16	8370	104 + 4, 108 − 2	107
20	8374	104 + 3, 108 − 2	106
26	8380	104 + 3, 108 − 2	106
Nov. 14	8399	104 − 2	102
20	8405	104 − 4, 99 − 1, 100 − 2	99
22	8407	104 − 4, 99 + 0, 100 − 1	99
28	8413	95 + 0, 104 − 5, 100 − 2, 99 − 2	97
Dec. 1	8416	100 + 2, 99 + 2	102
10	8425	100 + 0, 104 − 4	100
19	8434	99 + 0, 100 − 1	99
1882			
May 12	8578	107 + 2, 108 + 0	108
14	8580	107 + 3, 108 + 1	110
16	8582	107 + 3, 108 + 2	110
18	8584	107 + 3, 108 + 1	110
19	8585	107 + 3, 108 + 0	109
20	8586	107 + 3, 108 + 1	110
25	8591	107 + 3, 108 + 1	110
28	8594	107 + 3, 108 + 0	109
29	8595	107 + 3, 108 + 1	110
June 30	8627	108 + 3, 107 + 5	112
July 9	8636	104 + 6, 107 + 2, 108 − 1	109
24	8651	104 + 3, 108 − 3, 107 − 2	106
26	8653	107 − 3, 108 − 4, 104 − 1	104
29	8656	104 − 2, 108 − 4, 107 − 3	103
Aug. 19	8677	87 + 7, 95 − 3	93

TABLE XI.—*Baxendell's Observations of S Delphini*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1882			
Sept. 4	8693	87 + 4, 95 − 2	92
9	8698	87 + 5, 95 − 3	92
13	8702	87 + 4, 95 − 3	92
15	8704	87 + 4, 95 − 3	92
Oct. 5	8724	87 + 2, 95 − 3	90
28	8747	87 + 4, 95 − 2	92
1883			
July 28	9020	87 + 3	90
28	9020	87 + 4, 95 − 4	91
Oct. 28	9112	104 + 4, 108 − 1	108
1884			
May 18	9315	87 + 5, 95 − 3	92
24	9321	87 + 6, 95 − 2	93
26	9323	87 + 4, 95 − 1	92
27	9324	87 + 5, 95 − 2	92
30	9327	87 + 6, 95 − 2	93
June 11	9339	95 + 0	95
20	9348	95 − 1	94
25	9353	87 + 8, 95 + 0	95
July 2	9360	104 − 2	102
7	9365	99 + 3, 104 − 3	102
12	9370	104 + 0, 108 − 3	104
19	9377	104 + 3, 108 − 3	106
25	9383	104 + 5, 108 − 1	108
27	9385	108 − 1	107
Aug. 4	9393	104 + 5, 108 + 0	108
11	9400	108 + 2	110
15	9404	108 + 0	108
19	9408	108 + 2	110
23	9412	108 + 3	111
25	9414	108 + 3	111
Sept. 4	9424	108 + 2	110
11	9431	108 + 3, 107 + 3	110
16	9436	108 + 2	110
19	9439	108 + 2	110
20	9440	108 + 1	109
Oct. 9	9459	108 + 4	112
12	9462	108 + 2	110
1884			
Nov. 7	9488	104 + 5, 108 − 2	108
13	9494	104 + 1, 108 − 4	104
17	9498	104 + 0	104
27	9508	104 − 4	100
Dec. 3	9514	99 + 0, 104 − 6	98
6	9517	87 + 9, 100 − 3, 95 + 3	97
20	9531	87 + 7, 95 − 1	94
24	9535	87 + 10, 95 + 0	96
1885			
Jan. 5	9547	87 + 7, 95 − 1	94
8	9550	87 + 4, 95 − 2	92
11	9553	87 + 6, 95 − 2	93
14	9556	87 + 6, 95 + 1	94
May 7	9669	108 − 1	107
11	9673	108 + 0	108
June 9	9702	104 + 4, 108 − 1	108
July 4	9727	108 − 1	107
25	9748	108 − 1	107
Aug. 14	9768	108 − 1	107
20	9774	104 − 1, 108 − 4	104
28	9782	95 − 1	94
Sept. 3	9788	87 + 5, 95 − 2	92
9	9794	87 + 4, 95 − 2	92
14	9799	87 + 1, 95 − 5	89
27	9812	87 + 1	88
Oct. 29	9844	87 + 2	89
Nov. 8	9854	87 + 2	89
15	9861	87 + 3, 95 − 5	90
19	9865	87 + 3, 95 − 3	91
Dec. 1	9877	87 + 6, 95 − 1	94
26	9902	104 − 1, 108 − 4	104
1886			
Jan. 5	9912	104 + 2, 108 − 3	106
11	9918	104 + 0, 108 − 4	104
13	9920	104 + 2, 108 − 3	106
	241+		
May 21	0048	104 + 2, 108 − 2	106

TABLE XI.—*Baxendell's Observations of S Delphini*—continued.

Date.	J.D. 241+.	Corrected Comparisons.	Deduced Mag.
1886			
June 14	0072	95 + 3, 104 − 3	100
26	0084	104 − 2, 108 − 5	102
30	0088	95 + 0, 104 − 4	98
Sept. 14	0164	95 + 3, 104 − 3	100
Oct. 23	0203	104 + 4, 108 − 1	108
30	0210	108 − 1	107
Nov. 4	0215	104 + 5, 108 − 1	108
Nov. 18	0229	108 + 2	110
30	0241	108 + 3	111
Dec. 20	0261	108 + 3	111
25	0266	108 + 2	110
1887			
Jan. 12	0284	108 + 1	109
June 10	0433	87 + 3	90

Chandler's period is 277·5 days.

A period of 277·5 days having been assumed, the whole series was divided into portions of 23 or 24 days, seven of the former to one 24, taking the means of all the observations in each portion. Twelve consecutive portions are counted as one period.

The means of large groups of periods were then taken, as follows, in units of 0·1 magnitude.

Group.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
A	88	91	98	105	107	108	108	106	104	101	94	91
B	92	92	94	99	104	110	115	114	109	101	93	92
C	91	93	96	103	107	108	108	106	104	101	93	91
D	92	97	103	107	110	110	109	107	104	96	93	89

Group A contains periods I. to VI.; B, VII. to XII.; C, XXIV. to XXIX.; D, XXXI. to XXXVI.; the periods being numbered as in Table XII.

Chandler's formula, $2402621 + 277{\cdot}5E$, does not give any variation of period for this star, but inspection of the four "groups" given above suggests one with a cycle of between 40 and 50 periods, and a variation in length of period of ± 3 or 4 days. For it is evident that the first and third groups are identical in regard to position of maximum and minimum, showing that from period III. or IV. to period XXVI. or XXVII., the average length of period has been just about that which was assumed, viz. 277·5 days; while in the second group these places have *advanced* by nearly one "portion" (23 days), and in the fourth they have *receded* by about the same amount.

In order to obtain definite data, each period has been examined separately. First, a mean curve was formed, by shifting all the observations in group B one "portion" back, and all those in group D one "portion" forward, and then taking the mean of all the observations (including the isolated ones), thus obtaining the figures which are given at the foot of Table XII.

The curve thus formed is a very fairly smooth one, and its "upward" and "downward" slopes differ much less than usual. The magnitude from each observation was then compared with the

magnitude read off from the corresponding point of the curve, and the resulting "O–C" is given in Table XII.

TABLE XII.

Comparison of the Observations with a Mean Light Curve.

Period.	Initial Date. 240+	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
I.	0700	...	+4$_1$	+6$_2$	+2$_2$	...	...	...	...	+2$_2$	+ 1$_3$	−4$_5$	−4$_1$
II.	0977·5	−2$_5$	+1$_3$	+4$_3$	+2$_7$	0$_5$	−1$_5$	0$_2$	...	...	...	...	...
III.	1255	−1$_1$	...	+5$_1$	−1$_1$	+ 1$_4$	−2$_3$	−3$_5$	−1$_4$	+1$_4$	+ 4$_2$	+3$_2$	−1$_1$
IV.	1532·5	...	...	...	...	...	+4$_1$	0$_6$	−2$_4$	−2$_5$	0$_6$	+3$_2$	−1$_7$
V.	1810	−2$_7$	−2$_3$	+1$_4$	...	...	...	...	...	+2$_1$	+ 1$_3$	+3$_2$	+2$_4$
VI.	2087·5	−3$_3$	−4$_5$	−4$_7$	−2$_2$	0$_2$	...	...	...	...	...	...	...
VII.	2365	−1$_1$	−3$_5$	−6$_3$	−1$_1$	0$_3$	+3$_2$	+5$_4$	+5$_3$	+6$_4$	+ 6$_1$	+5$_1$	...
VIII.	2642·5	...	...	...	...	− 2$_3$	+2$_2$	+7$_2$	+8$_2$	+5$_2$	0$_3$	−2$_5$	+2$_5$
IX.	2920	+4$_3$	+3$_1$	+5$_2$	...	...	...	...	...	+4$_2$	+ 1$_1$	−3$_1$	−1$_3$
X.	3197·5	+2$_2$	+1$_2$	−5$_1$	−9$_1$	...	...	...	...	...	...	...	+1$_1$
XI.	3475	+1$_4$	0$_4$	−8$_4$	...	−10$_1$	−6$_1$	+5$_1$	+7$_2$	...	...	...	...
XII.	3752·5	...	...	...	...	− 4$_1$	...	...	...	...	...	+6$_1$	...
XIII.	4030	+2$_1$	...	...	...	...	...	...	...	...	...	...	...
XXIII.	6805	...	...	...	...	...	...	...	−1$_3$	...	...	...	...
XXIV.	7082·5	...	...	...	0$_4$	0$_3$	−2$_4$	−1$_4$	−1$_1$	+1$_4$	+ 2$_1$	+2$_3$	+2$_2$
XXV.	7360	+2$_3$	...	...	..	...	...	...	...	+2$_1$	+ 4$_5$	−3$_9$	0$_9$
XXVI.	7637·5	+2$_8$	+1$_9$	0$_3$	+1$_4$	...	...	...	...	+3$_4$	+ 1$_2$	−2$_3$	−2$_3$
XXVII.	7915	−1$_4$	−1$_6$	−2$_5$	+1$_3$	− 1$_3$	−4$_3$	−3$_2$	−1$_1$	...	...	...	...
XXVIII.	8192·5	−2$_2$	−2$_5$	0$_3$	−1$_3$	− 1$_3$	−1$_3$	−1$_3$	0$_4$	0$_2$	− 1$_4$	+4$_2$	...
XXIX.	8470	...	..	...	...	+ 1$_5$	+1$_4$	+3$_1$	−1$_3$	−6$_2$	− 6$_2$	−4$_3$	+2$_1$
XXX.	8747·5	...	...	...	...	...	...	...	...	...	...	...	0$_2$
XXXI.	9025	...	...	...	+3$_1$	...	...	...	...	...	...	...	...
XXXII.	9302·5	+2$_4$	+1$_3$	+3$_4$	+4$_4$	+ 3$_5$	+1$_5$	+2$_2$	...	0$_4$	− 4$_3$	0$_5$	...
XXXIII.	9580	...	...	...	+2$_1$	+ 2$_1$	−1$_1$	−2$_1$	−1$_1$	−3$_3$	−10$_3$	−9$_1$	−2$_2$
XXXIV.	9857·5	+1$_3$	+9$_1$	+7$_3$	...	...	...	...	...	+1$_1$	+ 1$_3$	...	...
XXXV.	241+ 0135	...	+8$_1$	+7$_1$	+5$_2$	+ 4$_2$	+1$_2$	0$_1$	...	...	...	...	...
XXXVI.	0412·5	−1$_1$	...	...	...	...	...	...	...	...	...	...	...
Mag. from curve		90	93	97	104	107	109	109	107	104	100	94	91
	$p-f$	−2	−7	−11	−10	−5	−2	+2	+5	+7	+10	+9	+4

The "correction to epoch" was then obtained for each period separately, using the observations individually, and weighting them by $(p-f)$. But as the maximum comes almost at the beginning of each period as arranged in Table XII., these corrections would be faulty as they stand; so the mean of several consecutive periods

has been taken instead, arranging them so that the epoch in question should come into about the middle of the observations used. As the periods are in most cases scantily observed, 4, 6, and even 8 periods have been used in obtaining these corrections to epoch, which are shown in the following table.

TABLE XIII.

Corrections to Epoch.

Period.	No. of Periods used in obtaining Mean.	Correction to Epoch (Days).	Correction to Period (Days).
II.	2	− 9·8	
			+5·3
III.	4	− 4·5	
			+5·5
IV.	6	+ 1·0	
			+3·6
V.	8	+ 4·6	
			+0·4
VI.	8	+ 5·0	
			+2·8
VII.	8	+ 7·8	
			+3·0
VIII.	8	+10·8	
			+2·9
IX.	8	+13 7	
			+1·8
X.	6	+15·5	
			(−13·8 in 15)
XXV.	2	+ 1·7	
			−0·1
XXVI.	4	+ 1·6	
			−3·2
XXVII.	6	− 1·6	
			−1·6
XXVIII.	4	− 3·2	
			−4·8
XXIX.	2	− 8·0	
			(−11·8 in 4)
XXXIII.	2	−19·8	
			−1·0
XXXIV.	4	−20·8	
			−0·9
XXXV.	2	−21·7	

It is clear that these figures bear out the conclusion tentatively formed from a consideration of the "groups" of periods, A, B, C, and D. When they are plotted on squared paper it becomes evident from inspection that (1) the true mean position of the epoch of maximum is nearly 3 days earlier than its position in the curve; (2) the half cycle of variation extends from about period 3·4 to about period 27·4, so that the full cycle would consist of 48 periods, and each period = 7°·5.

With these assumptions, the value of the amplitude of the curve of variation was obtained by calculation, each "correction to epoch" being weighted by the sine of the phase as well as by

the relative value of the data used in obtaining it. The mean result gave a value for C of a little over 24 days.

The formula required has therefore been assumed to be the following: *

$$2402645{\cdot}7 + 277{\cdot}5\mathrm{E} + 24 \cos (7^\circ{\cdot}5\mathrm{E} + 304^\circ{\cdot}5)$$

"E" being so taken as to coincide with the "E" of Chandler. The place of maximum in the mean curve was estimated at 6 days after the "starting" point, and this value is taken in the above formula, while 2·8 days are deducted for the difference between the epoch of the mean curve and the true mean position of the epoch as shown by the variation curve. The following Table shows the agreement between the epochs as "observed" and as calculated from the formula.

TABLE XIV.

Comparison of Epochs of Maxima.

	Baxendell. 240+	Calculated. 240+	B. – C. Days.	Chandler.	Bax. – Ch.
II.	0983·5 – 9·8	0980·7 – 4·4	– 2·6	0956·0	+ 17·7
III.	1261·0 – 4·5	1258·2 – 1·3	– 0·4	1233·5	+ 23·0
IV.	1538·5 + 1·0	1535·7 + 1·9	+ 1·9	1511·0	+ 28·5
V.	1816·0 + 4·6	1813·2 + 5·0	+ 2·4	1788·5	+ 32·1
VI.	2093·5 + 5·0	2090·7 + 8·0	– 0·2	2066·0	+ 32·5
VII.	2371·0 + 7·8	2368·2 + 10·9	– 0·3	2343·5	+ 35·3
VIII.	2648·5 + 10·8	2645·7 + 13·6	+ 0·0	2621·0	+ 38·3
IX.	2926·0 + 13·7	2923·2 + 16·1	+ 0·4	2898·5	+ 41·2
X.	3203·5 + 15·5	3200·7 + 18·2	+ 0·1	3176·0	+ 43·0
XXV.	7366·0 + 1·7	7363·2 + 7·4	– 2·9	7338·5	+ 29·2
XXVI.	7643·5 + 1·6	7640·7 + 4·4	0·0	7616·0	+ 29·1
XXVII.	7921·0 – 1·6	7918·2 + 1·3	– 0·1	7893·5	+ 25·9
XXVIII.	8198·5 – 3·2	8195·7 – 1·9	+ 1·5	8171·0	+ 24·3
XXIX.	8476·0 – 8·0	8473·2 – 5·0	– 0·2	8448·5	+ 19·5
XXXIII.	9586·0 – 19·8	9583·2 – 16·1	– 0·9	9558·5	+ 7·7
XXXIV.	9863·5 – 20·8	9860·7 – 18·2	+ 0·2	9836·0	+ 6·7
XXXV.	0141·0 – 21·7	0138·2 – 20·1	+ 1·2	0113·5	+ 5·8

* Some provisional calculations for this star were given in *Monthly Notices*, lxviii. p. 556: the formula found being

$$28{\cdot}3 \sin (7\tfrac{1}{2}^\circ\mathrm{E} + ?).$$

It is there pointed out that the correlation of the constants with period would assign the theoretical term

$$24{\cdot}7 \sin (7^\circ{\cdot}6\mathrm{E} + ?).$$

Miss Blagg arrived at the above figures

$$24 \cos (7^\circ{\cdot}5\mathrm{E} + 304^\circ{\cdot}5)$$

quite independently, having forgotten both the former computation and the correlated relations.—H. H. T.

The differences between Baxendell and Chandler are also shown in this Table; and in addition to the fluctuation they show a curiously large constant difference, of about one "portion" (*i.e.* one-twelfth of a period). This is a good illustration of the difficulties of estimating a maximum. This star appears to remain for about 60 days within 0·1 mag. of its maximum.

Although the "B–C" figures are extremely small, and very well distributed, yet the formula can of course be nothing more than a tentative suggestion, which may require considerable modification when tested by comparison with other observations. It would be possible, by assuming a general correction to the mean period, and by making different assumptions as to the length of the cycle and the place of the "mean" epoch, to form other formulæ which might fit Baxendell's observations fairly well.

The mean curve used in the reductions gives, when analysed, the following, expressed in magnitudes:—

$$0{\cdot}96\cos(\theta-169^\circ)+0{\cdot}10\cos 2(\theta-90^\circ)+0{\cdot}00\cos 3(\theta-?)$$

the arbitrary zero point of the curve being taken as the starting point. This is $11\frac{1}{2}$ days after the starting point of the period. The analysis gives "0" for the third term.

When divided by the range (1·98 magnitudes) and multiplied by 6, the expression becomes

$$2{\cdot}91\cos(\theta-169^\circ)+0{\cdot}30\cos 2(\theta-90^\circ).$$

This expression has the greatest negative value possible when $\theta=-7\frac{1}{2}^\circ$; and as 1° is equivalent to 0·77 days, this places the epoch of maximum in the curve at 5·8 days before the zero point, or 5·7 days after the starting point of the period, which is very close to the estimated 6 days fixed upon before the analysis was made.

Analysing the results for each group of periods (to 0·1 mag. only), we obtain the following figures, referred to the arbitrary zero:—

Mean curve	$2{\cdot}91\cos(\theta-169^\circ)+0{\cdot}30\cos 2(\theta-90^\circ)$
Group A [I.+(5)]	$2{\cdot}91\cos(\theta-170^\circ)+0{\cdot}68\cos 2(\theta-92^\circ)$
,, B [VII.+(5)]	$2{\cdot}92\cos(\theta-186^\circ)+0{\cdot}59\cos 2(\theta-19^\circ)$
,, C [XXIV.+(5)]	$3{\cdot}04\cos(\theta-171^\circ)+0{\cdot}33\cos 2(\theta-90^\circ)$
,, D [XXXI.+(5)]	$2{\cdot}89\cos(\theta-151^\circ)+0{\cdot}45\cos 2(\theta-63^\circ)$

The expressions for groups A and C are (as they should be) nearly identical with that for the mean curve, while those for B and D differ from it in the right direction as far as the larger term is concerned, and pretty nearly to the required amount. The range of magnitudes in the mean curve is 1·98; in A, 2 magnitudes; in B, 2·3; in C, 1·80; and in D, 2·1. The mean curve tends to be smaller in range than those formed from a less number of periods.

No. 9, T Delphini, called by Baxendell, S Delphini (7444 Chandler, 204016 Harvard).

$20^h\ 40^m\ 43^s\ +16^\circ\ 2'{\cdot}1$ (1900·0).
$20\ 38\ 38\ +15\ 52'{\cdot}5$ (1855·0).

TABLE XV.

Comparison Stars.

A.S.V. No.	A.S.V. Grad.	H.A. Letter.	H.A. Mag.	Bax.	Adopted Mag.
5	11	*b*	8·48	*a*	85
6	16	*c*	9·02	*b*	88
8	24	*d*	9·54	*c*	95
11	28	*f*	9·80	*d*	98
21	43	*k*	10·90	*e*	107
23	45		(11·08)	*f*	112
26	49		(11·50)	*g*	116
30	55	*m*	12·04	*h*	121
33	58	*n*	12·17	*k*	126

The H.A. magnitudes are from *Harvard Annals*. lvii. p. 265, those in brackets being inferred from *A.S.V.* grades. The Adopted Magnitudes were modified from Harvard to suit Baxendell's differences shown in Table XVI.

TABLE XVI.

Baxendell's Differences between his Comparison Stars.

Years.	88–85.	95–88.	98–95.	107–98.	112–107.	116–112.	121–116.	126–121.
1863–4	2_9	7_8	3_4	12_2	4_2	6_2	5_1	6_1
1865	...	...	4_6	9_2	...	...	10_1	2_1
1866–8	...	...	3_2	9_9	...	4_2	4_3	5_3
1878–9	...	....	...	12_2	...	4_1	6_1	6_3
1880–2	6_4	5_1	3_3	11_3	8_1	5_1	...	7_1
1883–4	...	14_5	5_6	14_7	7_1	4_3	...	...
1885–7	8_1	10_2	1_1	14_{12}	...	...	6_1	...
Means	3·6	9·4	3·6	12·1	5·8	4·6	5·6	5·3
No. of Comparisons	14	16	22	37	4	9	7	9
Reduced Means	3	8	3	10	5	4	5	5

The reduced means are obtained by applying the usual formula to the individual comparisons.

The telescopes used were A 68 until the end of 1869, and E 80 from 1877; except that R (13) 81 was used on 1865 Sept. 24 and on 1868 June 18; 22 (*a*) 39 was used on 1875 Dec. 5; and E was used with power 48 on 1877 Nov. 25 and Dec. 2, and with power 110 on 1877 Dec. 31.

TABLE XVII.

Baxendell's Observations of T Delphini.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1863			
Dec. 27	1867	126 + 0	126
1864			
Jan. 1	1872	126 + 2	128
May 16	2008	Not seen < 120	
26	2018	Invisible < 125	
June 6	2029	Not seen < 117	
16	2039	,, < 125	
July 4	2057	,, < 130	
8	2061	,, < 130	
13	2066	,, < 132	
26	2079	Suspected < 130	
29	2082	Just glimpsed 132	
Aug. 21	2105	98 + 1, 95 + 3, 88 + 7	97
23	2107	98 − 2	96
26	2110	88 + 5, 98 − 4	94
31	2115	88 + 3, 95 − 3	92
Sept. 5	2120	85 + 2, 88 − 1	87
9	2124	85 + 2, 88 + 0	88
11	2126	85 + 0, 88 − 2	86
14	2129	85 + 1, 88 − 1	86
19	2134	85 + 2, 88 + 0	88
24	2139	85 + 2, 88 + 0	88
26	2141	85 + 3, 88 + 1	88
Oct. 1	2146	85 + 4, 88 + 3, 95 − 3	91
3	2148	88 + 3, 95 − 2	92
5	2150	88 + 3, 95 − 3	92
8	2153	88 + 6, 95 − 1	94
9	2154	88 + 7, 98 − 2, 95 + 1	96
13	2158	88 + 9, 95 + 3, 98 + 0	98
17	2162	98 + 3, 107 − 7	100
28	2173	107 + 0	107
31	2176	107 − 2	105
Nov. 5	2181	107 + 1	108
Nov. 8	2184	107 + 6, 112 + 3, 116 − 2	114
23		Invisible < 114	
1865			
May 20	2377	Not seen < 130	
June 9	2397	,, < 115	
17	2405	,, < 130	
20	2408	,, < 130	
28	2416	126 + 3	129
July 1	2419	126 + 0	126
13	2431	107 + 0	107
14	2432	107 · 2	105
21	2439	98 + 0, 107 − 6	100
Aug. 3	2452	95 + 0, 98 − 3	95
12	2461	95 + 1, 98 − 3	96
15	2464	95 + 0, 98 − 4	94
25	2474	95 + 0, 98 − 3	95
31	2480	95 + 3, 98 + 0	98
Sept. 7	2487	98 + 1	99
23	2503	107 − 1	106
24	2504	107 − 1	106
27	2507	107 + 0	107
Oct. 2	2512	107 + 3	110
8	2518	107 + 6, 116 + 0	114
10	2520	116 + 1	117
19	2529	116 + 3	119
Nov. 7	2548	Glimpsed pretty steadily.	
12	2553	Occasionally glimpsed.	
15	2556	,, ,,	
21	2562	Not seen < 132	
1866			
May 17	2739	,, < 120	
20	2742	Suspected < 132	
24	2746	Invisible < 125	
June 8	2761	About 116 + 0	116
25	2778	98 + 0	98
July 11	2794	98 + 0	98

TABLE XVII.—*Baxendell's Observations of T Delphini*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1866			
July 16	2799	95 + 3, 98 − 1	98
20	2803	95 + 2, 98 + 0	98
22	2805	98 − 1	97
31	2814	98 + 1	99
Aug. 27	2841	107 + 1, 98 + 9	108
31	2845	107 + 2	109
Sept. 8	2853	121 − 3, 112 + 3	116
15	2860	121 + 3, 126 − 2	124
Oct. 1	2876	126 + 4	130
6	2881	Just glimpsed	132
9	2884	Just glimpsed occasionally	132
16	2891	Invisible	< 133
28	2903	Suspected.	
Dec. 30	2966	Invisible	< 132
1867			
Jan. 11	2978	Invisible	< 132
May 21	3108	98 + 3, 107 − 6	101
June 5	3123	98 + 2, 107 − 6	100
29	3147	98 + 2, 107 − 4	102
July 24	3172	98 + 2, 107 − 6	100
27	3175	98 + 2, 107 − 7	100
Aug. 1	3180	98 + 4, 107 − 2	104
6	3185	98 + 4, 107 − 2	104
23	3202	107 + 6	113
Sept. 2	3212	126 − 1	125
16	3226	126 + 1	127
24	3234	126 + 2	128
Nov. 1	3272	Not visible	< 127
1868			
May 20	3473	98 − 2	96
25	3478	98 − 2	96
30	3483	98 − 1	97
June 2	3486	98 + 0	98
4	3488	98 + 1	99
12	3496	98 + 2	100
15	3499	98 + 3	101
18	3502	98 + 3	101
1868			
June 19	3503	98 + 4, 107 − 3	103
26	3510	107 + 0	107
29	3513	107 + 0	107
July 10	3524	107 + 9	116
12	3526	112 + 5, 116 + 2, 121 − 2	118
13	3527	116 + 2, 121 − 2	118
23	3537	121 + 3, 126 − 1	124
25	3539	121 + 4, 126 − 1	125
Oct. 17	3623	Not seen	< 133
Nov. 8	3645	,,	< 133
15	3652	,,	< 130
1869			
June 8	3857	? Glimpsed	< 108
Oct. 26	3997	Not seen	< 133
1875			
Dec. 5	6228	Invisible	< 100
1877			
Nov. 25	6949	Not seen	< 120
Dec. 2	6956	,,	< 125
14	6968	,,	< 130
23	6977	,,	< 135
31	6985	,,	< 135
1878			
June 23	7159	107 + 0	107
29	7165	112 + 3	115
July 7	7173	112 + 3, 116 − 1	115
8	7174	116 + 0	116
12	7178	116 + 1	117
20	7186	121 + 0	121
21	7187	121 + 3, 126 − 4	123
27	7193	126 + 2	128
28	7194	126 + 0	126
Aug. 4	7201	126 + 3	129
6	7203	126 + 5	131
7	7204	126 + 3	129
18	7215	126 + 4	130
25	7222	126 + 5	131

TABLE XVII.—*Baxendell's Observations of T Delphini*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1878			
Aug. 28	7225	126 + 6	132
Sept. 2	7230	Invis. < 126 + 6	
15	7243	,, < 126 + 6	
24	7252	,, < 135	
Oct. 13	7271	,, < 126 + 0	
26	7284	,, < 126 + 7	
Nov. 1	7290	,, < 126 + 7	
Dec. 5	7324	,, < 126 + 0	
7	7326	,, < 126 + 0	
23	7342	Invisible.	
1879			
Jan. 13	7363	Invisible.	
19	7369	Invis. < 126 + 3	
Aug. 1	7563	,, < 107 + 11	
8	7570	,, < 126 + 6	
11	7573	,, < 126 + 6	
17	7579	< 126 + 7	
21	7583	< 126 + 7	
23	7585	Invisible.	
29	7591	,,	
Sept. 2	7595	Invis. < 126 + 4	
7	7600	,, < 126 + 7	
9	7602	< 135	
12	7605	Invisible.	
13	7606	Invis. < 126 + 8	
19	7612	,, < 126 + 7	
21	7614	,, < 126 + 8	
Oct. 4	7627	,, < 126 + 3	
14	7637	Invis. < 126 + 6	
15	7638	,, < 126 + 6	
17	7640	,, < 126 + 7	
20	7643	,, < 126 + 6	
Nov. 5	7659	,, < 126 + 7	
10	7664	,, < 126 + 7	
12	7666	Firmly glimpsed = 133	
Dec. 23	7707	121 + 0	121
24	7708	116 + 0	116
1880			
Jan. 2	7717	112 + 0, 116 − 4	112
Apr. 17	7823	116 + 0	116
29	7835	121 + 0	121
May 5	7841	121 + 1	122
12	7848	126 + 0	126
28	7864	Invis. < 126 + 6	
July 4	7901	< 135	
30	7927	< 135	
Aug. 6	7934	< 137	
Oct. 3	7992	Not seen < 136	
Nov. 20	8040	Glimpsed = 137	
21	8041	,,	
26	8046	About 133	
Dec. 1	8051	126 + 5	131
24	8074	107 + 2, 116 − 6	110
1881			
Jan. 6	8087	107 + 3	110
12	8093	107 − 3	104
May 7	8208	Invisible < 130	
20	8221	< 137	
29	8230	Not seen < 126 + 6	
July 19	8281	,, < 137	
27	8289	,, < 137	
Aug. 1	8294	< 137	
24	8317	< 137	
31	8324	< 137	
Sept. 14	8338	Glimpsed = 137	
19	8343	,, abt. 135	
25	8349	,, 135	
27	8351	,, 135	
Oct. 15	8369	126 + 3	129
16	8370	126 + 2	128
20	8374	121 + 3, 126 − 3	124
26	8380	116 − 1	115
Nov. 14	8399	98 + 3, 107 − 6	101
17	8402	98 + 3, 107 − 6	101
20	8405	98 + 1, 107 − 9	98
22	8407	95 + 1, 98 − 3	96

TABLE XVII.—*Baxendell's Observations of T Delphini*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1881			
Nov. 28	8413	98 − 4, 95 − 3, 88 + 1	92
Dec. 1	8416	85 + 5, 88 − 2	88
10	8425	85 + 4, 88 + 0	88
19	8434	85 + 6, 88 + 1	90
1882			
May 12	8578	Invis. <135	
14	8580	,, <135	
29	8595	Invisible.	
July 9	8636	Invisible.	
24	8651	Invis. <135	
Aug. 19	8677	<137	
Sept. 4	8693	Visible abt. 136	
9	8698	126 + 7	133
13	8702	126 + 8	134
15	8704	126 + 5	131
Oct. 5	8724	107 + 3, 112 − 4	109
28	8747	95 − 3, 98 − 5	92
1883			
July 28	9020	=133	
28	9020	(Later) about 133	
Aug. 25	9048	107 + 3, 121 − 5	113
Oct. 28	9112	98 + 4, 107 − 9	100
1884			
May 18	9315	Not seen <126 + 0	
24	9321	,, <116 + 0	
26	9323	<121 + 0	
27	9324	<116 + 0	
30	9327	<116 + 0	
July 2	9360	121 + 0	121
12	9370	116 − 1	115
19	9377	112 + 3, 116 + 0	116
23	9381	116 + 0	116
25	9383	116 − 2	114
27	9385	116 − 2	114
Aug. 1	9390	112 + 0	112
4	9393	107 − 3	104
11	9400	98 + 2, 107 − 9	99
1884			
Aug. 14	9403	98 + 1, 107 − 8	99
16	9405	98 + 2, 107 − 8	100
19	9408	98 + 2, 107 − 9	99
23	9412	98 + 0, 107 − 10	98
25	9414	98 + 2, 107 − 9	99
Sept. 4	9424	98 − 3	95
11	9431	88 + 10, 95 − 1, 98 − 7	94
16	9436	88 + 1, 98 − 7	90
17	9437	88 + 7, 98 − 5	94
19	9439	88 + 9, 98 − 6	94
20	9440	88 + 9, 98 − 4, 95 − 2	95
24	9444	88 + 9, 98 − 4, 95 − 3	94
Oct. 4	9454	88 + 8, 98 − 4, 95 + 2	96
9	9459	88 + 11, 98 − 3, 95 + 3	97
12	9462	95 + 2, 98 − 1	94
Nov. 7	9488	107 + 0	107
13	9494	107 + 6, 112 + 0	112
17	9498	112 + 5, 116 − 1	116
27	9508	112 + 4, 116 + 3	118
Dec. 3	9514	121 + 2	123
6	9517	121 + 4	125
1885			
Jan. 5	9547	Glimpsed 135	
May 7	9669	Invis. <126 + 6	
11	9673	,, <135	
June 9	9702	,, <135	
July 4	9727	116 + 2	118
9	9732	116 − 1	115
25	9748	98 + 9, 107 − 6	104
28	9751	107 − 5	102
Aug. 1	9755	98 + 7, 107 − 3	104
3	9757	98 + 7, 107 − 4	104
14	9768	98 + 6, 107 − 4	104

TABLE XVII.—*Baxendell's Observations of T Delphini*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1885			
Aug. 16	9770	98 + 7, 107 − 3	104
18	9772	98 + 9, 107 − 3	106
20	9774	98 + 7, 107 − 4	104
28	9782	98 + 8, 107 − 4	104
Sept. 3	9788	98 + 10, 107 − 3	106
9	9794	98 + 10, 107 − 2	106
14	9799	98 + 11, 107 + 0	108
27	9812	116 + 0, 121 − 5	116
Oct. 29	9844	126 + 5	131
Nov. 8	9854	126 + 7	133
Dec. 1	9877	< 136	
26	9902	< 135	
1886	241+		
May 21	0048	98 + 9, 107 − 2	106
June 5	0063	98 + 0	98
June 14	0072	85 + 7, 95 − 7, 88 + 0	89
26	0084	88 − 2	86
30	0088	88 − 2	86
Sept. 14	0164	Invis. < 130	
17	0167	126 + 4	130
Oct. 23	0203	Invis. < 126 + 3	
30	0210	,, < 126 + 4	
Nov. 4	0215	,, < 126 + 4	
18	0229	,, < 126 + 4	
30	0241	,, < 126 + 7	
Dec. 20	0261	,, < 126 + 4	
1887			
Jan. 12	0284	< 126 + 3	
June 10	0433	88 + 7, 98 − 5, 95 − 4	93

In the case of this star, as in that of R Delphini, Baxendell was unable to see it when near its minimum; and we find, on tabulating the observations in the usual manner, that more than one-third of the curve is unobserved. It is impossible, therefore, to analyse it harmonically; but the periods have been divided into groups, and the mean differences of each portion from the adopted curve are shown in Table XVIII. From these means, corrections to epoch have been found, for comparison with Chandler's formula, which is $2402133 + 331{\cdot}2E$. The epoch of maximum, read from the curve, seems to be about 273 days after the zero point.

TABLE XVIII.

Comparison of the Observations with a Mean Light Curve.

Periods.	Portions. 240+.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
I.	1860	+ 5	...	...	...	...	...	...	...	− 4	− 7	− 5	− 2
II.	2192	...	...	...	...	...	...	...	...	+ 8	− 2	− 1	− 1
III.	2524	− 1	...	...	...	...	...	...	...	+12	+ 1	+ 1	− 1
IV.	2856	+ 4	...	...	...	...	...	...	...	...	+ 4	+ 3	−10
V.	3188	− 6	− 3	...	...	...	...	...	...	...	...	− 3	− 6
VI.	3520	− 2	...	...	...	...	...	...	...	...	...	...	...
XVI.	6840	...	...	...	...	...	...	...	...	...	...	..	− 1

TABLE XVIII.—*Comparison of the Observations with a Mean Light Curve*—continued.

Periods.	Portions. 240+.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
XVII.	7172	− 2	0	...	...	...	...	...	...	...	...	...	...
XVIII.	7504	...	...	...	...	...	...	...	− 4	...	...	...	+ 5
XIX.	7836	+ 3	...	...	..	...	...	...	+15	+ 6	+ 9	...	...
XX.	8168	...	...	...	...	...	...	...	+ 3	− 8	− 7	...	...
XXI.	8500	...	...	...	...	...	...	...	+10	− 5	...	...	...
XXII.	8832	...	...	...	...	...	...	...	− 2	...	...	+ 4	...
XXIII.	9164	...	...	...	...	...	...	...	− 2	− 3	− 2	− 3	− 5
XXIV.	9496	− 3	...	...	...	...	...	...	...	+10	+ 8	+ 6	+ 7
XXV.	9828	+ 6	...	...	...	...	...	...	− 7	−10	−11	..	...
XXVI.	(0160)	+10	...	...	...	...	...	...	...	...	− 2	...	...
I.+(4)		0	−3	...	...	...	...	...	...	+ 5	− 1	− 1	− 4
XVII.+(4)		0	0	...	...	...	...	...	+ 6	− 2	+ 1	...	+ 5
XXII.+(4)		+ 4	...	...	...	...	...	...	− 4	− 1	− 2	+ 2	+ 1
Mag. from curve		124	131	...	...	...	...	130	120	105	96	98	110
$p-f$		−21	...	...	...	...	...	...	+25	+24	+ 7	−14	−26

The correction to epoch for the first group of periods is +6·3 days; that for the second group, −0·5 days; and that for the third, −5·7 days.

Mean E.	Baxendell.	Chandler.	Bax.−Ch.
2	2402797+6·3	2795·4	+ 7·9
18	8109−0·5	8094·6	+13·9
23	9769−5·7	9750·6	+12·7

These figures give a correction to period between the first and third group of −12·0 days in 21 periods, which would make the average length of the period during that time $= 332 - \frac{12}{21}$ days, or 331·4 days; 0·2 days longer than Chandler's.

But if we group together the first three and last three periods (which are among the best observed) we get the following results:—

Periods.	Mean E.	Baxendell.	Chandler.	Bax.−Ch.
I.+(2)	1	2402465+3·0	2464·2	+3·8
XXIV.+(2)	24	2410101−11·1	0081·8	+8·1

The maximum, as read from the curve, may well be a few days in error; but the above comparisons with Chandler suggest that either the period is really longer than he made it, or there should be a term similar to those which he gives in the case of other variables. Baxendell's observations are too scattered, and the resulting curve is too incomplete, to allow of any hope of finding from them its actual character.

A System of Photographic Magnitudes for Southern Stars.

By J. Halm, Ph.D.

(*Communicated by H.M. Astronomer, Cape of Good Hope.*)

The principal object of the following investigation is to establish a system of photographic magnitudes which may be used as a basis for the standardisation of the magnitudes of the Cape Astrographic Zones. As a first step in this direction it seemed desirable to examine particularly the relation between the system of magnitudes of the C.P.D. and a system based on accepted photometric principles, it being recognised that if a reliable reduction of the C.P.D. to such a system could be effected, the corrected C.P.D. magnitudes could be utilised with advantage and convenience as an intermediary for the conversion of the observed diameters of star-images in the Astrographic Zones into standard photographic magnitudes.

The derivation of such a system is dealt with in that part of the paper which contains an investigation of the photographic magnitudes of the C.P.D. stars within 1° of the South Pole.

In a subsequent discussion the derived magnitudes of these polar stars are then utilised for determining the corrections to the C.P.D. system in 24 areas distributed at nearly equal intervals along the parallel −45° Declination, the centres of these regions being arranged so as to agree approximately with those of Kapteyn's *Selected Areas*. The comparisons between the South Pole and these areas were effected by means of plates on which the Pole and one of the areas were photographed, as far as possible under the best and most uniform atmospheric conditions.

A third part of the paper deals with a provisional determination of the differences (photographic—visual magnitude) and the star's colour depending on the spectral type.

Theory and Method of Observation and Reduction. — The observational evidence, on which the determination of the photographic magnitude of a star is based, is naturally supplied by the actinic effect of its light on the photographic plate. If the latter is placed in the focus of the telescope, differences in the light intensity, either due to differences of intrinsic luminosity in the case of two stars of different magnitude or due to differences in the duration of exposure of one and the same star, are shown by corresponding variations in the size of the photographic images. The measurable quantity in this case is the diameter of the star disc.

An alternative method based on photometric measurements of the blackness of the *extrafocal* star-images was proposed and extensively employed by Professor Schwarzschild. Either of the two methods may be utilised in such a manner that the magni-

tude of any star on a given plate can be determined by means of an equation containing two parameters, whose values have to be ascertained from the photometric evidence afforded by the plate in question. This important point, already demonstrated by Schwarzschild and others, may be shown by the following consideration.

Let I_0 be the quantity of light of a star S_0 which falls upon the photographic plate in one second of time, and therefore $I_0 t$ the amount of light received on the plate during an exposure of t seconds. If the actinic effect is proportional to the intensity received, and if this effect is measured by the apparent diameter of the star disc d, we may write—

$$I_0 t = \phi(d) . \quad . \quad . \quad . \quad . \quad (1)$$

where $\phi(d)$ represents a so-far unknown function of the diameter. Suppose now t to be varied by impressing on the same plate a succession of images of different exposures, t_1, t_2 . . .; then writing (1) more conveniently in logarithmic form, we have a series of relations of the form—

$$\log I_0 + \log t_1 = \log \phi(d_1)$$
$$\log I_0 + \log t_2 = \log \phi(d_2) \ . \ . \ .$$
$$. \quad . \quad . \quad . \quad . \quad .$$

and from these the relation between the varying quantities d, as derived by direct measurement, and log t may be graphically represented.

On the other hand, let us fix our attention on one particular time of exposure, t_m. For any star S, of diameter d and intensity I, the quantity of light received on the plate is It_m. Suppose that this quantity corresponds exactly to the amount thrown on the plate by the star S_0 in time t_n, or that

$$It_m = I_0 t_n \ . \quad . \quad . \quad . \quad . \quad (2)$$

Now if the actinic effects were strictly proportional to the intensities received, the apparent diameters should agree. Hence the quantity log t_n could be found by entering the graph mentioned above with the diameter d.

Assuming, then, the intensity I_0 of the fiducial star S_0 to be known, the intensity I of any other star on the plate could be determined by means of equation (2).

The correctness of the assumption of strict proportionality between the received intensity and its actinic effect, however, has been negatived by the experimental investigations of Schwarzschild, Exner, Abney, and others. It is well known that if a source of light is exposed during a given time, the actinic effect is not exactly reproduced if the intensity is, for instance, doubled and the time of exposure halved. According to Schwarzschild, in order to

express equal actinic effects, equation (2) must be written in the modified form—

$$I t_m^p = I_0 t_n^p \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where p is an exponent entering into the problem as an additional unknown. Schwarzschild established the important fact that within very wide limits p may be considered independent of the time of exposure.

Considering the relation between the intensity I, and magnitude m, of a star—

$$I = A.10^{-0.4m},$$

equation (3) may be finally written in the form—

$$m = m_0 + 2.5p \log t_m/t_n \quad . \quad . \quad . \quad (4)$$

Here m_0 denotes the magnitude of the fiducial star S_0, t_m the time of exposure of star S, and t_n the time of exposure of star S_0 when it appears to have the same diameter as star S. Equation (4) contains two unknown quantities, m and p, so that at least two stars of known magnitude on a plate are required for the numerical determination of its coefficients.

It is readily seen that the same reasoning would have applied to the case in which the actinic effects are measured by Schwarzschild's method of extra-focal determinations of the densities of the images instead of focal diameters.

The photometric determination of m_0 on an absolute scale, *i.e.* the fixation of the zero-point of magnitudes, is outside the possibility of the photographic method. In the following investigation the value of m_0 is therefore based on *external* evidence, viz. the Harvard photometric magnitude of σ Octantis. As regards p, however, which defines the magnitude scale, a method can be devised by means of which this quantity may be determined from the *internal* evidence of the photographs, and therefore independent of any adopted photometric scale. Obviously what is required is the presence of two stars on the plate whose ratio of light intensity is accurately known. Such two or more objects may be optically produced by the introduction of a wire grating. The ratio of the intensities, or difference in magnitude, between the central and any of the diffracted images, is a quantity which permits of an accurate evaluation from the dimensions of the grating. Denoting by d and d' the measured diameters of the central and diffracted image (provided that the elongation of the latter due to colour dispersion is insensible), the corresponding values of $\log t_n$ and $\log t'_n$ may be obtained from the graph by the above method, and from (4) we get

$$p = \frac{0.4(m' - m)}{\log t_n - \log t'_n} \quad . \quad . \quad . \quad . \quad (5)$$

where $(m' - m)$ denotes the known difference of magnitude between the two images.

The application of the wire-grating method thus enables us to determine differences of photographic magnitude between any stars on the plate, so that, apart from fixing the zero-point of the magnitude scale, the problem of determining photographic magnitudes on a strictly photometric basis appears to admit of a practical solution by the above method.

Instruments and Appliances.—All the photographs were taken with the 13-inch Astrographic Equatorial, of which a full description is contained in Sir David Gill's *History of the Cape Observatory*, the method of observation being identical with that employed in the observation of the Astrographic Zones, except that no réseau lines were impressed on the plate. The wire grating placed in front of the object-glass was constructed in the workshop of the observatory. It consists of a square frame of cast aluminium with circular central opening which, by means of a clamp ring, can be attached to the tube of the telescope. Parallel and equidistant pianoforte steel wires of diameter 0·770 mm. are stretched across the frame in a manner similar to that used in pianos, so that their tension can be equalised by tuning them to uniform pitch. The ratio $\frac{a}{a+d}$ (a = width of open spacing, d = diameter of wire) was found to be 0·878, which, according to optical theory, corresponds to a difference of 4·34 magnitudes between the central and first diffracted image.

Measurement of Diameters of Star-Images.—As measuring apparatus a Repsold machine similar in construction to that used for the measurements of the astrographic plates was employed. The micrometer is provided with a double web, and pointings were made by placing successively the right-hand web against the left-hand limb and *vice versa*. The difference of the two micrometer readings thus represents the diameter of the image plus the distance between the two webs. Under good conditions the image of a star, provided sufficiently long exposure be given, consists of a fairly well-defined nucleus surrounded by a more or less diffuse halo or penumbra. Only in very faint objects, which however have not been included in the present discussion, the central nucleus is absent, and the images appear as ill-defined grey specks to which the diameter method cannot be applied. In measuring, the rule was strictly observed to set the web against the terminator of the central nucleus. With some practice the observer soon accustoms himself to a uniform habit of observing star discs of the same size in the same manner, and this uniformity of method is the only essential requirement for the method here proposed.

Reduction of Observations.—As an example for demonstrating the method of reduction, I select the measurements of plate xviii., on which nine exposures of the stars round the South Pole are photographed. The graphical relation between d and log t has been based on the mean of the two brightest stars on the plate, viz. σ Octantis (C.P.D. $-89°, 47$) and B Octantis ($-89°, 53$). The

following Table I. shows the corresponding values of d and log t for the nine exposures:—

Table I.

d.	log t.	d.	log t.	d.	log t.
r		r		r	
0·945	3·00	0·695	2·00	0·540	1·25
0·895	2·75	0·645	1·75	0·445	0·78
0·765	2·25	0·600	1·50	0·405	0·48

From the graph constructed from these numerical values we derive the next table of figures:—

Table II.

d.	log t.	d.	log t.
1·00	3·19	·65	1·77
·95	2·99	·60	1·53
·90	2·80	·55	1·30
·85	2·59	·50	1·06
·80	2·40	·45	·78
·75	2·20	·40	·46
·70	1·99	·35	·12

We now select as special examples of reduction the three stars, $-89°·47$, $-89°·37$, $-88°·40$.

Table III.

$-89°, 47.$				$-89°, 37.$				$-88°, 40.$			
Obs. d.	log t_m.	log t_n (Table II.).	$\frac{m-m_0}{2·5p}$.	Obs. d.	log t_m.	log t_n.	$\frac{m-m_0}{2·5p}$.	Obs. d.	log t_m.	log t_n.	$\frac{m-m_0}{2·5p}$
1·01	3·00	3·23	−0·23	0·70	3·00	1·99	1·01	0·45	3·00	0·78	2·22
0·95	2·75	2·99	−0·24	0·64	2·75	1·73	1·02	0·42	2·75	0·59	2·16
0·82	2·25	2·48	−0·23	0·53	2·25	1·20	1·05	0·34	2·25	0·05	2·20
0·77	2·00	2·28	−0·28	0·48	2·00	0·95	1·05				
0·71	1·75	2·03	−0·28	0·44	1·75	0·72	1·03				
0·65	1·50	1·77	−0·27	0·41	1·50	0·53	0·97				
0·59	1·25	1·48	−0·23	0·36	1·25	0·19	1·06				
0·47	0·78	0·90	−0·12								
0·44	0·48	0·72	−0·24								
		Mean	−0·24				1·03				2·19

m_0 here denotes the mean magnitude of the two stars on which Tables I. and II. are based.

For the evaluation of p the comparison between central and first diffracted image for $89°·47$ and $89°·53$ is shown in the following Table IV.:—

TABLE IV.

−89°, 47.					−89°, 53.				
Central Image.		1st Diff. Image.			Central Image.		1st Diff. Image.		
Obs. *d.*	log t_n (Table II.).	Obs. *d.*	log t'_n (Table II.).	log t_n/t'_n.	Obs. *d.*	log t_n (Table II.).	Obs. *d.*	log t'_n (Table II.)	log t_n/t'_n.
1·01	3·23	0·50	1·06	2·17	0·88	2·70	0·44	·72	1·98
0·95	2·99	0·46	0·84	2·15	0·84	2·54	0·39	·39	2·15
0·82	2·48	0·39	0·39	2·09	0·71	2·04	0·33	·00	2·04
0·77	2·28	0·36	0·19	2·09					
			Mean	2·12					2·06

The mean value of log t_n/t'_n is 2·096, and considering that $m - m_0 = 4{\cdot}34$, we obtain from (5)

$$p = 0{\cdot}828.$$

In order to convert the mean values at the foot of Table III. into magnitudes, they have therefore to be multiplied by the factor 2·07. Hence we obtain

−89°, 47	$m - m_0 = -0{\cdot}50$
−89°, 37	$= +2{\cdot}13$
−88°, 40	$= 4{\cdot}53$

For the determination of m_0 the Harvard photometric magnitude of σ Octantis (−89, 47) has been adopted. The visual magnitude of this star is 5·48. Its type being A_8, a correction $+0^m{\cdot}25$, according to Harvard, would be required for reduction to the photographic scale. Hence from the first of the preceding equations we have $m_0 = 6{\cdot}23$, and the photographic magnitudes of the other stars become

−89°, 37	photo. mag. = 8·36
−88°, 40	= 10·76

Comparison with the Potsdam Photometric System.—In order to test the important question how far the scale of the photographic magnitudes obtained by the above method agrees with that of standard photometric systems, such as the Potsdam system for instance, a special investigation of the brighter stars of the Pleiades was carried out. These stars have formed an important subject of investigation in Schwarzschild's photometric researches (*Publ. der v. Kuffner'schen Sternwarte*, Band v.). By adopting the Potsdam system in his determination of p, Schwarzschild's final magnitudes are naturally based on the scale of the system adopted, and a comparison of the Cape values with those found by Schwarzschild will therefore exhibit the differences between the standard Cape system as above defined and the Potsdam photometric system.

The mean results of the Cape magnitudes are based on the examination of three plates. All the stars mentioned by Schwarz-

schild were observed. It is sufficient here to summarise the result by combining the individual stars into groups arranged in order of magnitude. The constant m_0 has been so chosen that the mean difference (Cape – Schwarzschild) becomes zero.

TABLE V.

Group.	Stars.	Mean Magnitude of Group. Cape.	Schwarzschild.	Diff.
		m	m	m
I.	n, b, f, c, d,	3·98	4·07	−0·09
II.	e, h, g, k,	5·41	5·42	−0·01
III.	34, 1, p. 32, 38,	6·63	6·65	−0·02
IV.	12, 24, s, 19, 29,	7·16	7·13	+0·03
V.	4, 22, 10, 39, 17,	7·61	7·61	0·00
VI.	37, 31, 20, 23, 7,	7·97	7·97	0·00
VII.	33, 9, 1, 30, 18,	8·43	8·42	+0·01
VIII.	27, 13, 21, 2, 15,	9·03	8·95	+0·08
IX.	36, 8, 25, 11,	9·39	9·41	−0·02

The smallness of the differences shown in the last column points to a good agreement between the scales of the two systems.

Photographic Magnitude of 78 Stars near the South Pole.—In column 4 of Table VI. the photographic standard magnitudes of 78 stars in the immediate vicinity of the South Pole are exhibited. Their values are based on the results derived from ten plates, six of which were taken with the grating in front of the object-glass. Four earlier plates taken without the grating have also been utilised. It is obvious that only for the former set a value of p could be derived, but the systematic agreement between the values of $\log t_m/t_n$ shown in the 2nd and 3rd columns justifies the conclusion that the earlier observations are satisfied by the same value of p. In combining the two sets half weight was assigned to the earlier series.

TABLE VI.

C.P.D. No.	log t_m/t_n. Mean of 6 Plates with Grating.	4 Plates without Grating.	Photographic Magnitude.	C.P.D. (Hough).	Standard Photographic − C.P.D.
89°, 47	− ·25	− ·38	[5·73]	5·6	+ ·1
89, 53	+ ·25	+ ·27	6·85	6·7	+ ·1
89, 34	·64	·73	7·49	7·5	·0
89, 37	·98	1·11	8·41	7·7	+ ·7
88, 95	1·08	1·21	8·62	8·2	·4
89, 22	1·29	1·23	8·93	8·5	·4
88, 142	1·32	1·23	8·96	8·4	·6
89, 6	1·37	...	9·13	8·6	·5
89, 3	+1·59	+1·62	9·60	8·9	+ ·7

TABLE VI.—*continued.*

C.P.D. No.	log t_m/t_n. Mean of 6 Plates with Grating.	4 Plates without Grating.	Photographic Magnitude.	C.P.D. (Hough).	Standard Photographic − C.P.D.
89°, 5	+1·65	...	9·70	9·2	+ ·5
89, 33	1·59	+1·76	9·70	8·9	·8
88, 46	1·69	1·70	9·78	8·8	1·0
89, 50	1·87	1·82	10·11	9·2	·9
89, 11	1·96	1·95	10·34	9·3	1·0
89, 12	1·99	2·04	10·44	9·5	·9
88, 133	2·06	2·01	10·50	9·6	·9
89, 42	2·06	2·05	10·54	9·5	1·0
88, 132	2·07	2·09	10·58	9·7	·9
89, 52	2·10	2·08	10·60	9·6	1·0
89, 51	2·12	2·05	10·62	9·6	1·0
89, 16	2·14	2·13	10·71	9·6	1·1
88, 136	2·11	2·22	10·73	9·6	1·1
89, 38	2·16	2·17	10·75	9·5	1·2
88, 138	2·22	2·16	10·83	9·9	·9
88, 85	2·22	2·17	10·83	9·8	1·0
88, 118	2·22	2·23	10·87	9·8	1·1
88, 40	2·23	2·23	10·89	9·7	1·2
89, 43	2·27	2·14	10·89	9·7	1·2
89, 19	2·25	2·25	10·93	9·6	1·3
89, 35	2·22	2·33	10·95	9·8	1·2
89, 24	2·21	2·36	10·95	9·9	1·1
89, 26	2·27	2·37	11·03	9·9	1·1
89, 56	2·29	2·31	11·04	9·7	1·3
88, 119	2·34	2·41	11·20	10·0	1·2
89, 39	2·32	2·34	11·10	9·7	1·4
89, 21	2·40	2·38	11·22	10·0	1·2
88, 80	2·40	2·38	11·22	10·0	1·2
88, 89	2·41	2·40	11·26	10·1	1·2
89, 48	2·46	2·34	11·28	10·1	1·2
88, 91	2·45	2·43	11·32	10·1	1·2
88, 114	2·41	2·53	11·34	10·2	1·1
89, 29	2·43	2·48	11·34	9·9	1·4
89, 40	2·47	2·44	11·36	9·9	1·5
89, 23	2·47	2·44	11·36	10·0	1·4
89, 9	2·47	2·48	11·38	10·0	1·4
89, 14	+2·47	+2·49	11·40	10·1	+1·3

TABLE VI.—*continued.*

C.P.D. No.	log t_m/t_n. Mean of 6 Plates with Grating.	log t_m/t_n. 4 Plates without Grating.	Photographic Magnitude.	C.P.D. (Hough).	Standard Photographic − C.P.D.
89°, 57	+2·48	+2·50	11·42	10·1	+1·3
88, 144	2·55	2·47	11·49	10·3	1·2
89, 10	2·51	2·54	11·49	10·0	1·5
89, 31	2·55	2·58	11·57	10·2	1·4
89, 7	2·58	2·56	11·59	10·1	1·5
89, 30	2·57	2·59	11·61	10·2	1·4
89, 45	2·65	2·51	11·65	10·3	1·4
89, 44	2·63	2·54	11·65	10·3	1·4
89, 49	2·63	2·63	11·71	10·2	1·5
89, 4	2·64	2·62	11·71	10·1	1·6
89, 20	2·65	2·56	11·69	10·2	1·5
89, 61	2·65	2·62	11·73	10·4	1·3
88, 121	2·68	2·60	11·75	10·7	1·0
89, 25	2·66	2·65	11·77	10·4	1·4
89, 18	2·67	2·67	11·79	10·4	1·4
89, 54	2·68	2·68	11·81	10·3	1·5
89, 28	2·69	2·65	11·81	10·2	1·6
80, 60	2·67	2·72	11·83	10·5	1·3
89, 27	2·69	2·73	11·85	10·4	1·5
88, 73	2·71	2·60	11·79	10·5	1·3
89, 15	2·72	2·76	11·92	10·5	1·4
89, 58	2·76	2·71	11·94	10·5	1·4
89, 55	2·71	2·80	11·94	10·5	1·4
88, 37	2·74	...	11·94	10·4	1·5
89, 13	2·75	2·76	11·96	10·5	1·5
89, 59	2·77	2·78	12·00	10·7	1·3
89, 41	2·79	2·73	12·00	10·4	1·6
89, 36	2·71	2·88	12·00	10·6	1·4
89, 8	2·79	2·78	12·04	10·5	1·5
89, 46	2·81	2·80	12·06	10·4	1·7
89, 17	+2·80	+2·80	12·06	10·4	+1·7

The value of p used for the conversion of the figures of columns 2 and 3 into differences of photographic magnitude is derived from the comparison of the central and first diffracted images of the three brightest stars − 89°, 47, − 89°, 53, and − 89°, 34. The individual results for each of the six plates are given in the following Table VII. The meaning of the two last columns of this table will be explained later on.

TABLE VII.

Observed Values of p.

Plate.	p.	Diam. for Longest Exposure taken from Graph.	Computed p.
VIII.	0·796	1·05	0·809
IX.	0·847	0·99	0·827
X.	0·785	1·17	0·773
XVII.	0·838	0·95	0·839
XVIII.	0·828	0·95	0·839
XIX.	0·830	0·96	0·836
Mean	0·821±0·007	1·01	0·821

The 5th column in Table VI. contains the C.P.D. magnitudes; they are not those actually given in the C.P.D., but represent the values derived by Mr. Hough in his discussion of 917 Circumpolar Stars (*Cape Annals*, vol. xi., part ii.). Although slightly differing from the original values given in the C.P.D., so far as individual stars are concerned, they may be considered to represent generally the C.P.D. system. Finally, the last column of Table VI. shows the differences between the Cape photographic magnitudes here derived and the C.P.D.

Examination of the Variability of the Exponent p.—From equation (4) it follows that if the magnitudes of the same stars—supposed to undergo no intrinsic alterations in their light emission—are derived from different plates, changes of p, dependent on the particular conditions of the individual observations, should become evident by corresponding changes in the quantities $\log t_m/t_n$ derived from each plate separately. That such changes are actually taking place has been shown in Schwarzschild's investigations of the extra-focal images. It would appear, from the evidence of the present work, that their existence is less clearly shown by the diameter method. What variations there are seem to be of a more or less accidental character, so that comparing groups of four to six plates with each other, we meet with a remarkable constancy of p or $\log t_m/t_n$ from group to group. Taking, for instance, the differences in the values of $\log t_m/t_n$ in columns 2 and 3 of Table VI. in groups arranged in order of magnitude, we find:

TABLE VIII.

Mean Mag. m	No. of Stars.	Log tm/tn.
6·7	3	+0·01
8·9	4	−0·03
9·7	3	−0·07
10·7	20	−0·01
11·7	45	+0·01

The figures shown in the last column exhibit no evidence of a progressive change with magnitude, and hence point to the con-

clusion that essentially the same value of p satisfies the two sets of plates.

The same question has been examined somewhat more closely by a discussion of the results of the individual plates in the earlier of the two sets (column 3, Table VI.), more especially with the view of ascertaining how far p depends on the brand of plate employed. The result of this examination is shown in Table IX. where the differences of log t_m/t_n between the individual plate and the mean of the four plates are shown for four groups of stars arranged in order of magnitude.

TABLE IX.

Plate.	Mean Mag.	Δ log t_m/t_n.			
		$7^{m}\cdot5$	$9^{m}\cdot5$	$10^{m}\cdot5$	$11^{m}\cdot5$
9624	Ilford Monarch	+ ·02	− ·02	− ·01	+ ·03
9628	,,	− ·06	·00	+ ·03	+ ·02
9629	Flashlight	+ ·01	+ ·03	·00	− ·02
9630	Special Rapid	+ ·04	− ·01	+ ·03	− ·02

Here, again, no very marked changes progressive with magnitude are noticeable. It may be mentioned that the same developer (Amidol) was used in all cases, but that the duration of development was varied.

In the later series the values of p exhibited in Table VII. range between 0·785 and 0·847. It is interesting to note a somewhat close correspondence between the size of the star-images and the value of p. It is a well-known fact that the same star exposed during the same time on different plates produces images of different size. If, for instance, we take from the graphs for each individual plate the diameter corresponding to the longest time of exposure (log $t=3\cdot00$), we find the values shown in the 3rd column of Table VII., which may be taken to indicate the changes from plate to plate in the size of the images of the same star and same length of exposure. It will be noticed at once that the smallest values of p correspond with the largest diameters, and *vice versa*. If we compute p from the equation

$$p = 0\cdot821 - 0\cdot30\ (d - 1\cdot01),$$

d representing the figures shown in the 3rd column, the numerical values of the last column of Table VII. are obtained. By a correction of this nature the p.e. of p for a single plate is sensibly reduced, viz. from ±0·017 to ±0·009.

A similar correspondence between the changes of p and those of the density of the extra-focal star-image from plate to plate is noticeable in Schwarzschild's investigation. In his paper (*l.c.*, p. 48, Table 6) Schwarzschild has published the values of p derived from eighteen extra-focal photographs of the Pleiades. For the same plates the estimated degrees of density, σ, for the mean of the five brightest stars and exposure 81^{s}, were computed from

Table 3 of his paper, and the corresponding values of p and σ were then combined into three groups of plates arranged in order of increasing values of p, as follows:—

p.	σ.	p comp.
0·780	10·9	0·781
0·820	9·6	0·818
0·886	7·2	0·885

The figures of the 3rd column, computed from the linear relation

$$p = 0{\cdot}829 - 0{\cdot}028\,(\sigma - 9{\cdot}2),$$

are in remarkably close agreement with the values actually observed, and also the mean values of p for the two sets, Cape and Schwarzschild, are practically identical.

Much more material and more systematically conducted experiments are required before the suspected dependence of p on the observed actinic effects can be considered as established, and till then it may perhaps be wise to refrain from an attempt at explanation. Schwarzschild has already pointed out that the effect of "definition" should be expected to work in the opposite direction from what is here observed, and the results in Table VII. seem to support Schwarzschild's contention.

But whatever the causes of this phenomenon may be, they appear to be of such a nature that their effects became evanescent, if groups of plates are inter-compared. The general result of the present investigation seems to point to the conclusion that the photographic magnitudes of the polar stars shown in Table VI. would be systematically reproduced if the same method were applied to any set of photographs taken with different instruments and at different observatories.

As a further proof of this assertion, two older sets of observations made at Potsdam may be quoted. One of these, consisting of two photographs of the Pleiades taken with the Astrographic Equatorial, is mentioned by Scheiner in his book, *Photographie der Gestirne*, on p. 229. It consists of two plates taken under different conditions of definition, the material being published in such a form that the quantities $\log t_m/t_n$ may be derived by the above method. These quantities, for both the Potsdam and Cape observations, are shown in Table X. for four groups of stars, of which the designations in letters and figures (according to Bessel) and mean magnitudes (Schwarzschild) are given in the first two columns.

Table X.

Groups.	Mean Mag.	$\log t_m/t_n$.	
		Potsdam.	Cape.
	m		
g, k, 34, p, 1	6·26	0·00	0·00
12, 24, 29, 19	7·14	0·41	0·40
22, 4, 33, 20	7·80	0·71	0·76
30, 1, 9	8·41	1·01	1·03

By addition of a constant the values of log t_m/t_n for the brightest group have been reduced to zero. The figures of columns 3 and 4 show very good agreement throughout the whole range of magnitudes, thereby demonstrating that the Potsdam and Cape series are satisfied by the same factor p. Finally, the data referring to a single plate taken with a 5-inch Steinheil refractor and published by Scheiner in *Ast. Nach.*, No. 2884, were discussed in a similar manner, with the following result:—

TABLE XI.

Groups.	Mean Mag.	log t_m/t_n.	
		Potsdam.	Cape.
η, f, b, c	3·95	0·00	0·00
e, d, h, g	5·04	0·46	0·53
p, 34, 24	6·66	1·29	1·31
38, 10, 19, 22	7·31	1·64	1·64
9, 18, 23, 7, 30, 1	8·35	2·07	2·10

The fact that observations taken under so widely different conditions are satisfied by essentially the same value of p, is of great importance for photographic photometry. Supposing that the constancy of p can be relied on, the method here discussed will enable us to find the photographic magnitudes of all the stars on any plate, even without the use of a grating, provided that the magnitude of only one star on that plate is known from other sources.

Standardisation of the C.P.D. Magnitudes in Zone − 45°.—The photographic magnitudes of the stars of the south polar area shown in the 4th column of Table VI. may now be utilised for the determination of magnitudes in other parts of the sky.

If both areas are photographed on the same plate, and if precautions are taken that the atmospheric conditions in both cases are as nearly alike as is possible in practice, the stars of the polar area afford the means of establishing the relation between their diameters and magnitudes, and from the graph representing this relation the magnitude of any star of the other area may be directly found by entering the curve with its observed diameter and noting the corresponding magnitude.

In this manner 24 areas, whose centres are shown in the subjoined Table XIII., were examined. The photographs of area and of pole were taken on the same plate at equal altitude, two exposures of 20 and 6 minutes being made. For the present purpose it was considered sufficient to select 34 polar stars, excluding a number of the fainter stars and particularly those at large distances from the centre of the plate. The stars retained enable us to establish the desired relationship between diameters and magnitudes throughout the whole range of magnitudes (5·7—12·1). The examination of the stars in the −45° areas was limited to those for which C.P.D. magnitudes are known. Here,

again, the selection of stars was limited to about 40 in cases where the total number of C.P.D. stars exceeded that figure.

Having ascertained the standard photographic magnitudes of these stars in the 24 areas by the above methods, the results were compared with the C.P.D. magnitudes.

Considering first the mean result of this comparison obtained from a combination of all the areas, Table XII. exhibits the differences (Standard – C.P.D.) for nine groups whose mean photographic magnitudes are shown in the 1st column.

Representing these differences by equations of the form

$$\text{Diff. (Standard} - \text{C.P.D.)} = a(m - m_0)$$

where m represents the mean photographic magnitude of the group, a least square solution leads to the values

$$a = 0^{\cdot}450\,; \quad m_0 = 7^{\cdot}54.$$

The computed differences are exhibited in the 4th column of Table XII.

TABLE XII.

Mean Photographic Mag.	Observed Diff. (Standard – C.P.D.).	No. of Stars.	Computed Diff. (Standard – C.P.D.).	Obs.–Comp.
m	m		m	m
7·40	−0·15	55	−0·06	−0·09
8·50	+0·49	81	+0·44	+0·05
9·29	+0·86	98	+0·79	+0·07
9·78	+1·04	94	+1·01	+0·03
10·22	+1·22	131	+1·22	0·00
10·66	+1·37	114	+1·40	−0·03
11·06	+1·53	117	+1·58	−0·05
11·46	+1·73	120	+1·76	−0·03
11·93	+2·00	187	+1·98	+0·02

The observed differences (Standard – C.P.D.) are surprisingly well represented by the linear relation—

$$\text{Standard} - \text{C.P.D.} = 0^{\cdot}450(m - 7^{\text{m}}{\cdot}54).$$

We may therefore conclude that, at least in the zone here considered and between the 7th and 12th magnitude, the magnitudes of the C.P.D. represent a true photometric system whose scale value, however, is considerably too small.

The same conclusion is arrived at if the areas are individually examined, with the modification, however, that the scale correction undergoes sensible fluctuations depending on the Right Ascension of the area.

In order to demonstrate the character of these fluctuations, the differences (Standard – C.P.D.) for the extreme magnitudes $5^{\text{m}}{\cdot}0$ and $12^{\text{m}}{\cdot}2$, as derived from the material in each area, are collected

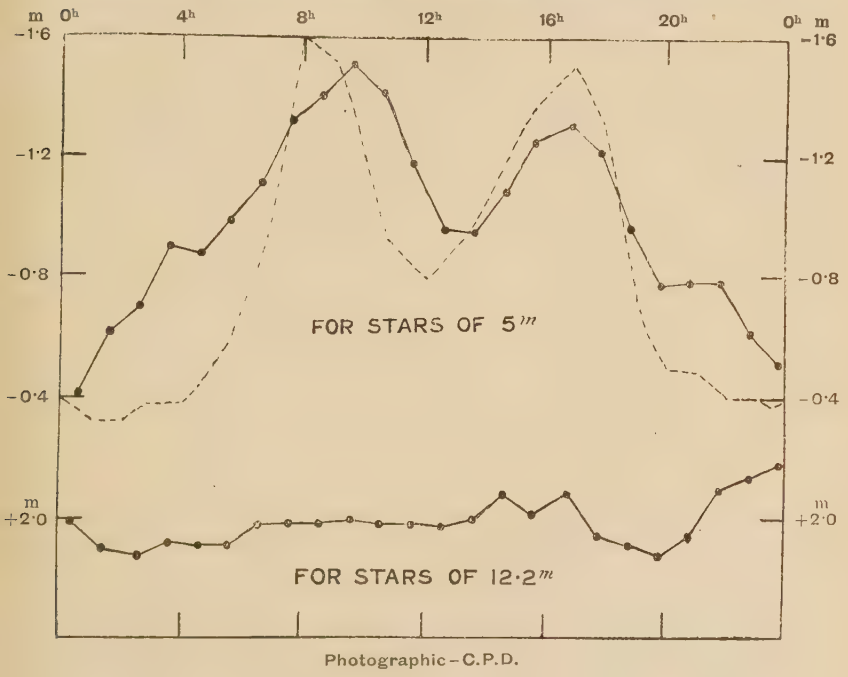
in the 2nd and 3rd column of Table XIII. It will be noticed that, while the changes depending on R.A. relating to faint stars are comparatively small and bear the character of accidental rather than systematic errors, a very pronounced fluctuation with two maxima and minima is shown in the figures pertaining to the bright stars. The character of this oscillation becomes, of course, somewhat more pronounced if the figures are "smoothed" by combining three neighbouring values into one, as shown in the 4th and 5th columns.

TABLE XIII.

Area.	Standard – C.P.D.		Smoothed.	
	$5^m{\cdot}0$.	$12^m{\cdot}2$.	$5^m{\cdot}0$.	$12^m{\cdot}2$.
h m				
0 35	−0·72	+2·04	−0·41	+1·98
1 35	−0·25	+1·76	−0·64	+1·89
2 35	−0·94	+1·88	−0·69	+1·86
3 35	−0·87	+1·94	−0·90	+1·92
4 35	−0·90	+1·93	−0·86	+1·91
5 35	−0·82	+1·86	−0·98	+1·91
6 35	−1·22	+1·93	−1·11	+1·97
7 35	−1·28	+1·96	−1·33	+1·97
8 35	−1·50	+2·01	−1·40	+1·97
9 35	−1·42	+1·93	−1·51	+1·99
10 35	−1·62	+2·04	−1·41	+1·94
11 35	−1·19	+1·84	−1·18	+1·93
12 45	−0·74	+1·91	−0·96	+1·96
13 45	−0·94	+2·07	−0·93	+1·99
14 45	−1·12	+1·93	−1·08	+2·07
15 45	−1·18	+2·16	−1·25	+2·01
16 45	−1·44	+1·95	−1·33	+2·07
17 45	−1·37	+2·11	−1·23	+1·94
18 45	−0·89	+1·75	−0·96	+1·91
19 55	−0·62	+1·86	−0·76	+1·86
20 45	−0·78	+1·98	−0·79	+1·94
21 45	−0·98	+1·97	−0·78	+2·09
22 45	−0·59	+2·33	−0·61	+2·14
23 45	−0·26	+2·14	−0·52	+2·17

As a general result derived from the figures of Table XIII., we may assert that the corrections required to reduce the C.P.D. magnitudes to the photographic system here adopted are fairly uniform in all Right Ascensions of zone -45° for stars of the 12th magnitude, but undergo pronounced changes in the case of bright stars. Bright stars of the same photographic magnitude are estimated in the C.P.D. considerably fainter at 9^h and 17^h than, for

instance, at 0^h Right Ascension. In the figure the numerical values of Table XIII. are graphically exhibited. The dotted curve represents the star density on the parallel -45° taken from the statistical investigations of Stratonoff. An intimate connection between the differences (Standard – C.P.D.) and the position of the area in relation to the Milky Way is undeniable. Considering that the C.P.D. system, although photographic, is essentially based on the visual observations of the Cordoba Zones, the fluctuations shown in the values (Standard – C.P.D.) are *de facto* systematic variations in the differences (photographic – visual). Interpreted



from this point of view, the result of Table XIII. means that bright stars of the same visual magnitude are *actinically* brighter in those regions of the -45° parallel which are intersected by the Galaxy than near the galactic poles. The reality of this phenomenon will be corroborated in the following section by observations made elsewhere.

Comparison of the C.P.D. with the Harvard Photographic System.—In a paper on "Spectra and Photographic Magnitudes of Stars in Standard Regions" (*Annals of Harvard College Observatory*, vol. lxxi., No. 2), Miss Fleming has published for certain standard regions (nine of which are centred on the -45° parallel and R.A. $1^h\ 20^m$, 4^h, $6^h\ 40^m$, $9^h\ 20^m$, $12^h\ 0^m$, $14^h\ 40^m$, $17^h\ 20^m$, $20^h\ 0^m$, $22^h\ 20^m$) the photographic magnitudes of those stars whose

spectra have been classified. These magnitudes are based on visual photometric observations which are converted into photographic magnitudes by means of corrections dependent on the type of spectrum. The visual Harvard and the C.P.D. magnitudes are also given. It is therefore possible to compare the C.P.D. system with the Harvard photographic system for the region of the sky discussed in the previous section.

Collecting first, as we did before, the complete material for all hours of R.A. into groups in order of magnitude, we find the results shown in the first two columns of Table XIV., the 3rd column containing the number of stars included in each group.

TABLE XIV.

Mean Photographic Mag. of Group.	Harvard Photographic minus C.P.D.	No. of Stars.	Harvard – C.P.D. Computed.	Obs.–Comp.
m	m		m	
5·24	−0·92	22	−0·91	−0·01
6·61	−0·47	34	−0·36	−0·11
7·49	+0·11	56	−0·01	+0·12
8·45	+0·42	46	+0·37	+0·05
9·26	+0·70	33	+0·69	+0·01
9·75	+0·84	36	+0·89	−0·05
10·36	+1·11	26	+1·13	−0·02

By a least square solution we find the differences of the second column to be represented by the numerical relation

$$(\text{Harvard} - \text{C.P.D.}) = 0{\cdot}399(m - 7{\cdot}52),$$

if for m the values of the 1st column are substituted. The differences (Harvard photographic – C.P.D.) computed from this formula are given in the 4th column, while the 5th column shows the differences between the observed and computed values. By the smallness of the residuals the comparison with Harvard confirms our previous conclusion that the C.P.D. magnitudes can be reduced to a photometric standard system by suitable corrections to its zero-point and scale value.

From the two relations—

$$(\text{Cape Standard} - \text{C.P.D.}) = 0{\cdot}450(m - 7{\cdot}54)$$
$$(\text{Harvard photographic} - \text{C.P.D.}) = 0{\cdot}399(m - 7{\cdot}52)$$

we derive finally

$$\text{Cape Standard} - \text{Harvard photographic} = +0{\cdot}051\ (m - 7{\cdot}5),$$

the p.e. of the factor being $\pm 0{\cdot}014$.

There is a fairly distinct indication of a systematic difference between the scale values of the Cape and Harvard system. It is important to note that a similar difference has been pointed out

by Seares* between the Mount Wilson and Harvard scales. He finds

$$\text{Mount Wilson} - \text{Harvard} = +0^{m}\cdot37 + 0\cdot070\,(m - 11\cdot0)$$
$$= +0\cdot070\,(m - 5\cdot7).$$

The Cape observations indicate therefore a scale correction to the Harvard system in the same direction and of nearly the same amount as the Mount Wilson series.

It was pointed out in the preceding section that the scale of the C.P.D. as compared with the Cape Standard system shows pronounced fluctuations depending on the R.A. of the region. Considering that the C.P.D. magnitudes, although obtained from photographs, are systematically based on visual observations, viz. on the Cordoba estimates of brightness, there is reason to think that the observed fluctuations are due to differences between the photographic and visual magnitude scale in various parts of the sky. In order to test this assumption, the differences Harvard (photographic – visual) were more minutely examined. In Table XV. these differences are given for each of the standard regions in -45° declination, first for the mean of all stars brighter than $8^{m}\cdot5$, and secondly for those fainter than that magnitude.

TABLE XV.

R.A. of Region.	Diff. (Photographic – Visual) for Stars brighter than $8^{m}\cdot5$.	Diff. (Photographic – Visual) for Stars fainter than $8^{m}\cdot5$.
h	m	m
1·3	+0·64	+0·92
4·0	+0·67	+0·73
6·7	+0·33	+0·94
9·3	+0·19	+0·58
12·0	+0·50	+0·86
14·7	+0·47	+0·52
17·3	+0·16	+0·59
20·0	+0·66	+0·66
22·7	+0·59	+0·76

The figures for the brighter stars show very distinct indications of the fluctuations with a principal and subsidiary maxima at 1^{h} and 13^{h} respectively, and two minima at about 9^{h} and 17^{h}, entirely in accordance with what was found in the preceding section for the bright stars of the Cape regions. The figures relating to the faint stars show very little of a systematic character, thereby also confirming the previous conclusion regarding the faint stars of the Cape areas. Bearing in mind the fact that the Harvard photographic magnitudes were derived from the visual magnitudes simply by the addition of corrections for "type," the present result indicates that in the zone -45°, among the bright stars, early types are more prevalent in regions near the Milky Way than in other parts of the zone, whilst faint stars are more uniformly distributed as regards type or colour.

* "The Photographic Magnitude Scale of the North Polar Sequence," Contributions from the Mount Wilson Solar Observatory, No. 70.

Magnitude in Relation to Position of Star on Plate.—The dependence of the size of the star-image on its distance from the centre of the plate has been examined by investigating separately within the 24 areas the differences (Cape Standard—C.P.D.) for three groups of stars situated at various distances from the centre. The results are shown in Table XVI., the figures indicating the deviations in magnitude in each group from the mean of the three groups.

TABLE XVI.

Mean Mag.	Distance from Centre.		
	0′–25′.	25′–45′.	>45′.
m	m	m	m
7·4	−0·02	−0·01	+0·02
8·5	−0·06	+0·02	+0·05
9·3	−0·09	+0·02	+0·06
9·8	−0·02	−0·01	+0·03
10·2	−0·07	−0·01	+0·07
10·7	+0·03	−0·01	−0·02
11·1	+0·04	+0·05	−0·10
11·5	+0·01	0·00	−0·01
11·9	+0·01	+0·03	−0·03

There are indications pointing to the conclusion that bright stars appear somewhat fainter at the margin of the plate than near the centre, and faint stars somewhat brighter. Peculiarly noticeable is the abrupt change of sign between $10^m{\cdot}2$ and $10^m{\cdot}7$.

Probable Error.—If on each plate the differences (Cape Standard − C.P.D.) are represented by the relation : diff. $= a + bm$, and the values computed from this formula are deducted from the observed differences, the residuals may be taken to represent the combined effect of the accidental errors inherent in the present observations and in the C.P.D. magnitudes, not including systematic plate errors. It has been found that the probable error of a single difference (Cape Standard − C.P.D.) formed in this manner amounts to $\pm 0^m{\cdot}10$. According to Kapteyn's discussion of the p.e. of a C.P.D. magnitude, the latter quantity amounts to $\pm 0^m{\cdot}08$. Hence the p.e. of a magnitude of the present Cape system determined from one plate is found to be $\pm 0^m{\cdot}06$. The result speaks for the high accuracy of the magnitude determinations of the C.P.D. and for their reliability after reduction to the Cape photographic system, for forming a basis for the determination of the magnitudes of the Astrographic Zones.

Relation between Photographic Magnitude and Colour of Star.—The method of the wire grating permits the determination of the wave-length of maximum intensity from the angular distance between the centre of the direct star-image and the point of maximum density in any of the diffracted images.

If we denote this distance, expressed in seconds of arc, by D, the order of the diffracted image by n, the distance between the centres of two adjoining wires of the grating by a, and the wave-

length of maximum intensity by λ_m, we may accept with sufficient accuracy the formula for normal incidence

$$aD \sin 1'' = n\lambda_m,$$

a relation from which λ_m can be determined.

A tentative investigation limited to 30 bright stars of different spectral types was undertaken mainly in order to test the accuracy of the method. The adopted procedure was to photograph with one exposure a group of stars of various types on one plate, on which also a succession of different exposures of σ Octantis was taken. The latter enables us to determine the photographic magnitudes of these stars, while the measurements of the relative distances of the different images from the central image afford means for ascertaining the wave-lengths of maximum intensity. As the stars had to be observed at different altitudes, the effect of extinction had to be taken into account. For this purpose a table based on observations by Wirtz and published in *Ast. Nach.*, vol. cliv. p. 361, was used. The visual photometric magnitudes and spectral types were taken from the Revised Harvard Photometry.

An example may illustrate the method of observation and reduction. On Plate XI. the micrometer readings of the diffracted images of σ Octantis were observed as shown in column 2 of Table XVII., the successive orders being taken from left to right—

TABLE XVII.

Order n.	Micrometer Reading. r	Obs.–Comp. r
5 left	7·660	+0·007
4 ,,	8·094	−0·004
3 ,,	8·534	−0·010
2 ,,	8·981	−0·008
1 ,,	9·439	+0·004
1 right	10·337	+0·012
2 ,,	10·775	+0·004
3 ,,	11·219	+0·003
4 ,,	11·654	−0·008
5 ,,	12·103	−0·004

Assuming that the micrometer-readings of the 2nd column are represented by equations of the type

$$R = R_0 \pm nD,$$

D denoting the distance between two consecutive images expressed in revolutions, and R_0 being the micrometer-reading of the centre, we find from a least square solution

$$R_0 = 9^r{\cdot}880,\ D = 0^r{\cdot}4454 \pm 0^r{\cdot}0005,$$

and the p.e. of a single setting $\pm 0^r{\cdot}0050$.

The numerical relation between D expressed in revolutions of the micrometer screw and λ_m (λ_m being expressed in tenth metres) is

$$\lambda_m = 9700\ D.$$

Hence

$$\lambda_m = 4320 \pm 5.$$

The p.e. is determined from the inter-agreement of the individual readings and does not include errors due to changes from plate to plate. From the individual values of λ_m for five plates we find the p.e., including plate error, to be ± 8.

Altogether five plates were taken, each plate containing single exposures of 6 stars of different types and a series of exposures of σ Octantis. A sixth plate, also containing 6 "type" stars, but without the last-named reference star, was also included in the determination of λ_m. The results are shown in Table XVIII. arranged in groups according to spectral type.

TABLE XVIII.

Name of Star.	Type.	Derived Photo. Mag. m	λ_m.	No. of Plates.	Visual Mag. Harvard. m	Photo.—Visual Mag. m
ν Eridani	B_2	4·21	4270	2, 1	4·12	+ ·09
δ Fornacis	B_5	4·85	4247	1, 1	4·93	− ·08
υ^1 Eridani	B_9	3·47	4282	2, 1	3·59	− ·12
τ^9 Eridani	A_p	4·98	4266	1, 1	4·69	+ ·29
α Doradus	A_p	3·44	4281	2, 1	3·47	− ·03
σ Octantis	A_8	(5·73)	4333	5	5·48	(+ ·25*)
δ Horologii	F_1	5·17	4304	1, 1	4·85	+ ·32
o^1 Eridani	F_5	4·34	4339	2, 1	4·14	+ ·20
γ Doradus	F_5	4·36	4337	1, 1	4·36	·00
τ^6 Eridani	F_8	4·65	4335	1, 1	4·33	+ ·32
o^2 Eridani	G_5	5·02	4358	1, 1	4·48	+ ·54
α Reticuli	G_5	4·02	4381	2, 1	3·36	+ ·66
y Eridani	K_1	5·49	4411	1, 1	4·58	+ ·91
δ Eridani	K_1	4·20	4438	1, 1	3·72	+ ·48
β Reticuli	K_1	5·07	4432	1, 1	3·80	+1·27
g Eridani	K_1	5·32	4400	1, 1	4·24	+1·08
α Horologii	K_1	4·93	4404	2, 1	3·83	+1·10
45 Eridani	K_1	5·47	4469	1, 1	4·97	+ ·50
υ^1 Eridani	K_1	5·55	4409	1, 1	4·59	+ ·96
υ^2 Eridani	K_1	4·98	4402	1, 1	3·88	+1·10
l Eridani	K_1	5·61	4408	1, 1	3·98	+1·63
h Eridani	K_2	5·67	4458	1, 1	4·64	+1·03
ϵ Reticuli	K_2	5·30	4432	1, 1	4·42	+0·88
γ Eridani	K_5	4·56	4489	2, 1	3·19	+1·37
ι Reticuli	K_5	6·28	4469	1, 1	4·81	+1·47
d Eridani	K_5	5·14	4482	1, 1	4·06	+1·08
π Eridani	M_a	5·61	4492	1, 1	4·64	+0·97
δ Reticuli	M_a	6·15	4471	1, 1	4·41	+1·74
54 Eridani	M_a	6·38	4421	1, 1	4·54	+1·84
γ Reticuli	M_b	6·04	4466	1, 1	4·46	+1·58

* Adopted from Harvard corrections for "type."

The figures of this table show undoubtedly systematic changes in λ_m and in the difference (photographic - visual), which are progressive with the type of the stars. It is interesting to compare the results with those obtained by other observers. As regards the difference between photographic and visual magnitude, Table XIX. contains a comparison between the group values of the final column of Table XVIII. and the magnitude corrections for "type" of the Harvard photometry.

TABLE XIX.

Type.	(Photographic – Visual).		
	Cape.	Harvard.	Cape – Harvard.
	m	m	m
B_5	−0·04	−0·17	+0·13
A_3	+0·17	+0·10	+0·07
F_5	+0·21	+0·51	−0·30
G_5	+0·60	+0·91	−0·31
K_1	+1·00	+1·17	−0·17
K_4	+1·17	+1·38	−0·21
M	+1·53	+1·68	−0·15

There is slightly greater range in the Harvard corrections than in those of the Cape, evidenced by a progressive "run' in the differences of the last column. But on the whole the agreement may be considered satisfactory, taking into account the small number of stars included in the present investigation.

With regard to the values of λ_m a most valuable series of observations based on the same method was published by Bergstrand in his paper "Recherches sur les couleurs des étoiles fixes, Upsala 1909." Bergstrand's definitive values of λ_m arranged in groups of spectral types, and the group values of the 4th column of Table XVIII., are shown in Table XX.

TABLE XX.

Bergstrand.		Cape.	
Type.	λm.	Type.	λm.
B_5	4165	B_5	4266
A_1	4230	A_3	4293
F_8	4280	F_5	4329
K_1	4436	G_5	4370
M	4497	K_1	4419
		K_4	4466
		M	4463

Here, again, the range in the Cape observations seems to be smaller than that evidenced by Bergstrand's results, the discrepancy being chiefly noticeable in the early types; but in view of the fewness of the Cape observations no importance can as yet be attached to

these discrepancies. From the figures of Tables XIX. and XX. relating to the Cape observations we derive the following relation between the photographic and visual magnitudes as function of λ_m:

$$\text{Photographic} = \text{visual} + 0^{m}\cdot00713\ (\lambda_m - 4280).$$

The internal agreement between the values of λ_m within the same type (Table XVIII.) compares favourably with that obtained by Bergstrand. Determining, for instance, the p.e. of λ_m for stars of type K_1, we find for Cape ± 15, for Bergstrand ± 40. It is worthy of note, in this connection, that Bergstrand used a grating of comparatively large dispersion, and determined the distances between the maximum points of the low order spectra, whereas in the present work a grating of comparatively low dispersion was employed which, however, admitted the measuring of the distances of spectra of higher order.

Summary.—(*a*) The photographic magnitudes of the 78 south polar stars contained in Table VI. may be considered to conform with the conditions required in a truly photometric system.

(*b*) A comparison of these magnitudes with the Harvard photographic system through the intermediary of C.P.D. stars in -45° declination points to a correction to the Harvard magnitude scale in satisfactory agreement with that found by Seares at Mount Wilson.

(*c*) The system of the C.P.D. may be reduced to a standard photographic system of magnitudes by corrections to its zero-point and scale value; one magnitude in the C.P.D. corresponding, in the average of stars over all Right Ascensions, to 1·82 of a magnitude in the Cape system and 1·67 in the Harvard system.

(*d*) The scale value of the C.P.D. in -45° declination shows considerable fluctuations, apparently depending on a relation of the stars to the Milky Way, and explainable by the fact that bright stars in that particular region of the Galaxy which intersects the parallel -45° are on the average of earlier spectral type than in other parts of the zone.

(*e*) Observations of stars of different types confirm qualitatively, and very nearly quantitatively, the differences (photographic – visual) found by the Harvard observers.

(*f*) In accordance with the results of Bergstrand and other observers, the wave-lengths of maximum intensity are found to be a function of the spectral type, the earlier types showing smaller wave-lengths than the later ones. The amplitude of this variation appears to be smaller in the Cape results than in those derived by Bergstrand.

Royal Observatory, Cape of Good Hope:
1914 *April* 15.

On the Nebular Line λ3729. By J. W. Nicholson, M.A., D.Sc., Professor of Mathematics in the University of London.

At the last meeting of the Society, Mr. J. H. Reynolds exhibited a slide, obtained with a prismatic camera at the Helwan Observatory, which extended to the Orion nebula the results already obtained by Dr. Max Wolf, in the case of another nebula, for the line λ3729. These results have been emphasised in earlier papers, and Mr. Reynolds' observations complete the practical demonstration that λ3729 has a different chemical origin from other lines such as λ5007, and that the nebulæ must contain at least one other element in addition to nebulium. In a paper on the constitution of nebulæ, presented by the writer at the same meeting, a complete theoretical scheme of these elements was indicated, although the particular element which gives rise to the line λ3729 was not specified. It is an element whose chief spectral lines are almost entirely in the ultra-violet, beyond the range of possible detection by astrophysical observations, and although its nature can be found with some degree of certainty by means of the universal relation between the angular momenta of atoms of different elements, given in detail in the *Monthly Notices* of 1914 January, a single link in the chain of reasoning was incomplete. Instead of putting forward the theoretical explanation of this line, it seemed better to wait until further experimental work was at hand. The writer was accordingly unwilling, in the discussion of Mr. Reynolds' paper, to express a definite opinion as to the origin of λ3729, until some further evidence could be found capable of proving or disproving, from an experimental standpoint, the origin which the theory suggests. This evidence, which is of a confirmatory character, has now appeared, for in the *Comptes Rendus* of 1914 April 6, MM. Bourget, Fabry, and Buisson have published some results which have ensued from the application to nebular spectra of the interference method devised by Fabry and Buisson themselves. The main object of this method is to determine the mass of the atom which emits a spectral line. A very complete and critical account of it has been given recently by Dr. Capstick in the *Observatory*,* and we shall therefore only make a very brief statement of its essential principles.

If an atom at rest could emit a vibration of a definite period, the corresponding spectrum line should be monochromatic or infinitely narrow. But the atoms of a gas—or molecules if it is in a molecular state—are not at rest, being endowed with the motions studied in the Kinetic Theory. If m and v are the mass and average velocity of an atom or molecule,

$$\tfrac{1}{2}mv^2 = a\tau,$$

where τ is the absolute temperature, and a is a universal constant

* 1913 May 1.

of all gases. A single atom has a velocity which varies in magnitude and direction according to the impacts which it experiences, but the average velocity is in all cases proportional to $\sqrt{\frac{\tau}{m}}$, the constant of proportionality being known accurately from the work of Lord Rayleigh and Schönrock. These atomic or molecular velocities produce Doppler effects on the emitted lines. The greatest effects are produced by velocities directly away from or towards the observer, and are in opposite directions on the scale of wave-lengths, so that the line is broadened on either side.

In the absence of any other broadening agency, the width of the line is therefore proportional to $\sqrt{\frac{\tau}{m}}$. If v is the extreme velocity of an atom, and λ the wave-length of the centre of the line,—λ therefore belonging to the stationary atom,—the increase of wave-length produced when the atom is moving directly away in the line of sight is given by $\delta\lambda = \lambda v/c$, where c is the velocity of light. Thus

$$\frac{\delta\lambda}{\lambda} \propto \sqrt{\frac{\tau}{m}}$$

and $2\delta\lambda$ is the total width of the line.

Such widths are too small for direct observation, and Buisson and Fabry obtain them by an indirect method. Strictly monochromatic light can interfere for any difference of path between two components into which it can be separated. But if it is not monochromatic, being spread instead over a narrow range of wave-length, the positions of reinforcement for the two components of one extreme wave-length will ultimately coincide with those in which the other extreme components produce minima, if the difference of path is steadily increased. Fringes cannot then be observed. The number of complete mean waves in the difference of path when this occurs is called the order of interference N, and if the breadth of the line is $2d\lambda$, and its mean wave-length λ,

$$N = \frac{\lambda}{2d\lambda}.$$

If, therefore, the broadening of a line is due only to the heat-motions of the atoms or molecules, $N \propto \sqrt{m/\tau}$.

Bourget, Buisson, and Fabry have observed N for the line $\lambda 3729$ in the Orion nebula, and also for the hydrogen lines. Since the temperatures of the mixed gases may be assumed to be equal, the square of the ratio of the two values of N is the ratio of the atomic masses of the particles from which the lines proceed. The actual numbers indicated approximately by the experiments are:—

For the line $\lambda 3729$, $N = 16{,}500$.
For the hydrogen lines, $N = 10{,}000$.

Their ratio is 1·65, and its square is 2·72. This is the approximate atomic weight of the element emitting the line $\lambda 3729$,

for the hydrogen lines have been proved already to proceed from atoms and not molecules. The authors conclude finally that the atomic weight of this unknown element is probably very nearly equal to 3. Visual experiments only have been performed on the line λ5007 of nebulium. This line is to be investigated more thoroughly in later experiments, but the visual work has indicated that the atomic weight of nebulium is probably greater than that of hydrogen and smaller than that of the element producing λ3729.

The fundamental importance of these results requires no emphasis. For, as theory has already predicted by methods which are of a rather more speculative character, the nebulæ evidently contain substances of a simpler nature than helium, whose atomic weight is 4, and these substances give the most characteristic lines in their spectra. Any precise or quantitative theory, according to which the nebulæ occupy the initial place in an order of stellar evolution, is therefore strengthened, especially since the only two terrestrial elements for which there is direct evidence in nebulæ are hydrogen and helium, the two of the simplest character. In fact, the nebulæ must be composed of a set of chemical forms, which are the simplest forms in which matter can exist. Elements are involved which are not in the Periodic Table. A favourite theory of the nebular spectrum has postulated hitherto that such lines as λ5007 and λ3729 are in some way due to hydrogen and helium. This theory cannot remain in the presence of a precise determination of atomic weights. The modern chemical theory of isotopes, suggested by the behaviour of radioactive products, and according to which all so-called elements are really mixtures of "isotopic" elements of somewhat different atomic weights, is also inapplicable, for the spectra of isotopes, so far as evidence has yet gone, are essentially the same. Nebulium, for example, cannot be an isotope of hydrogen or helium, for it has a very different spectrum. In other words, it is not a form of hydrogen or of helium, but a new element lying between them in the Table. The general conclusion must be that in the nebulæ are contained the elements which can fill an upper row in the Table, preceding helium. These are the atomic structures from which all the heavier elements have been derived by an evolutionary process, which must be the exact converse to the process of radioactivity. But it has been shown very clearly that the modifications of atoms which occur in radioactive phenomena consist only in the expulsion of α and β particles from the elementary nuclei. We can deduce, therefore, that stellar evolution proceeds by a progressive modification of the atomic nuclei.

This point of view, which is the logical outcome of the now demonstrated existence of new elements of very small atomic weight in the nebulæ, is, of course, the view which has been urged, on purely spectral grounds, by the writer in various papers, and more particularly in the paper communicated at the last meeting of the Society, where a complete scheme of the elements in

nebulæ was given. It is now necessary to find the place which Bourget, Buisson, and Fabry's atomic weights find in that scheme. The theoretical atomic weight of nebulium is 1·31, as deduced from the accurately measured lines λλ5006·89 and 4363·50 in its spectrum. That of protofluorine, from the coronal lines, is almost exactly 2, and that of the element with nucleus $+2e$—the origin of the characteristic nebular line λ3868·88—is 0·3275. These values satisfy the ratios

$$0{\cdot}3275 : 1{\cdot}31 : 2 :: 2^2 : 4^2 : 5^2$$

extremely closely, as in *M.N.*, 1914 January, and show that the masses of simple nuclei are proportional to the squares of their charges. We can deduce the atomic weights of the other simple-ring elements. For the element with nucleus $+6e$, which will be called *Archonium*, and which has already been indicated * as the ultimate nebular source of the ultra-violet radiation λ3484 of Wolf-Rayet stars, the atomic weight is

$$1{\cdot}31 \times \frac{6^2}{4^2} = 2{\cdot}947.$$

The next element, with nucleus $+7e$, has an atomic weight

$$1{\cdot}31 \times \frac{7^2}{4^2} = 4{\cdot}01,$$

practically coincident with that of helium. It is evident that the only simple element with an atomic weight appropriate to that found by the three observers is *archonium*, for 2·95 is as close to 3 as the accuracy of their method requires. The experiments thus indicate that λ3729 is produced by the element whose nucleus is $+6e$, or archonium, and this is the elementary origin to which the writer was formerly led by the general relation between simple-ring atoms in respect to angular momenta.

The neutral atom of archonium was dealt with in the last paper, but the consideration of its charged configurations has been postponed, for the reason specified already. They were, however, treated in an Adams Prize essay more than a year ago. It is sufficient, for the present, to indicate the particular charged atom which gives rise to the line λ3729. This is the atom which has lost two electrons, and which therefore retains four. The previous paper showed that the chief lines due to the neutral atom are at λλ2613·6, 2178·3, and therefore beyond the visible limits, so that the presence of archonium in the nebulæ was deduced solely from the fact that one of its neutral products is manifested in Wolf-Rayet stars. The same remarks are applicable to the atom which has lost one electron, but when it loses two, both the principal lines enter the visible region, and they are both well known in nebulæ. Wright's accurate measurements give them the wave-lengths 4068·8 and 3729·0. Their ratio is 1·09113, the last figure

* *Monthly Notices*, 1914 April.

being only approximate, in view of the order of accuracy of Wright's observations as tested by the hydrogen lines.

If they correspond to the vibrations $q = \omega$ and

$$q = \omega \sqrt{\nu - \tfrac{1}{4}S_n\left(1 + \frac{n}{2}\frac{m}{M}\right)}$$

of an atom for which $\nu = 6$, $n = 4$, their ratio should be, since $\frac{1}{4}S_4 = 0{\cdot}9571$,

$$\sqrt{\frac{6}{5{\cdot}0429}}\left(1 + 2\frac{m}{M}\right).$$

But in nebulium $\frac{m}{M} = 0{\cdot}000414$, and in archonium this ratio is smaller in the ratio $\frac{4}{9}$, becoming $0{\cdot}000184$. The ratio of the wave-lengths should therefore be

$$\sqrt{\frac{6}{5{\cdot}0429}}(1{\cdot}00037) = 1{\cdot}09116.$$

The difference is only 3 parts in 100,000, within the order of accuracy of the best determinations of wave-lengths in the nebulæ. The provisional identification of λ4068·8 given in the earliest paper on nebulium has been known to be incorrect since Dr. Bromwich's correction for the nucleus made it possible to estimate the wave-lengths to the fraction of a tenth metre.

These are the only important lines of archonium which manifest themselves in the ordinary nebular spectrum, although there are weaker ones which sometimes become important in the later stages of novæ. Since, of these two lines, λ4068·8 is comparatively inconspicuous, the line λ3729 becomes the only striking representative of this element. Its isolated character in the spectra of nebulæ is therefore not surprising.

MM. Bourget, Buisson, and Fabry are continuing their observations on the line λ5007. Their preliminary results are consistent with the theoretical value 1·31 for the atomic weight of nebulium. Observations on such a line as λ3869 would be more difficult, as it is much weaker, but they would, if successful, supply an even more searching test of the theory, for the element giving this line has an atomic weight only about one-third of that of hydrogen.

We may, in conclusion, indicate a very extraordinary result which the observations have given—that the temperature of the Orion nebula may be 15,000° C. This is deduced from the degree of broadening of the hydrogen lines, but the authors point out that it only constitutes an upper limit of temperature, for any cause tending to diminish the clearness of the interference fringes will lead to a smaller value of N, and a larger final value for the temperature. The result must therefore be accepted with as much caution as the authors themselves show. If the value were nearly correct, serious difficulties would arise with regard to the physical

condition and even the permanence of the nebula. But such sources of confusion in the fringes will operate equally on the fringes formed from hydrogen lines and from the archonium line $\lambda 3729$; and since the approximate atomic weight of the latter element is deduced by a method of ratio which eliminates the temperature, this atomic weight at least may be accepted with some confidence, especially in view of one feature of the method. In its previous application to terrestrial elements, it has been shown to be very satisfactory in the case of monatomic elements such as helium or neon. There is no evidence that molecules, as distinct from atoms, exist in nebulæ—in fact, the evidence is to the contrary. This absence of molecules, which is rendered probable by the simplicity of the hydrogen spectrum in nebulæ, is an important reason for confidence in the results, in so far as they relate to atomic weight.

The Resolution of a Compound Periodic Function into Simple Periodic Functions. By J. B. Dale, M.A.

1. The Fourier analysis of a compound periodic function assumes that we know both the period within which the complete fluctuation takes place, and also the entire history of the function during the period. If the history, given in the form of a table or graph, be not known for the entire period, still, provided that the period is known, the coefficients of the different harmonics can be found with no more difficulty than that involved in solving a set of simultaneous simple equations. When, however, the period is unknown, the problem of resolving the function into its component simple periodic functions presents considerable difficulty, not a small part being due to the limited degree of accuracy with which the values of the function are known. Professor Schuster has shown that periods may be discovered by means of a Periodogram, and more recently Professor Turner (*M.N.*, vol. lxxiii. p. 716) has shown that the changes of sign in the terms of a Fourier series may lead to the same end. These methods are, however, somewhat indirect.

The object of the present paper is to show that the fundamental periods of a periodic function may be found in terms of the roots of an equation of the nth degree, n being the number of fundamental periods, and the coefficients being functions of $3n+1$ equidistant values of the function. A method for the discovery of the number of periods is also given.

The Equation for the Periods.

2. We shall suppose the independent variable to be t, and the value y of the function to be known for the values 0, 1, 2, . . . $3n$ of t. We then write

$$y = A + \sum_{r=1}^{r=n} (B_r \cos(\theta_r t) + C_r \sin(\theta_r t)) \quad . \quad . \quad (1)$$

where θ_r is the angle by which the argument changes in the unit of t. If we introduce exponential values of the sine and cosine, putting $exp(i\theta_r) = x_r$, the equation becomes

$$y = A + \sum_{r=1}^{r=n} (D_r x_r^t + E_r x_r^{-t}) \quad . \quad . \quad . \quad (1')$$

Indicating the different values of y by the suffixes 0, 1, 2, . . . $3n$, we have the equations

$$\begin{aligned} y_0 &= A + \Sigma(D_r + E_r) \quad . \quad . \quad . \quad . \quad (2) \\ y_1 &= A + \Sigma(D_r x_r + E_r x_r^{-1}) \\ y_2 &= A + \Sigma(D_r x_r^2 + E_r x_r^{-2}) \\ y_k &= A + \Sigma(D_r x_r^k + E_r x_r^{-k}). \end{aligned}$$

. . . . . .

Subtract each of these equations in order from the following, then

$$\begin{aligned} y_1 - y_0 &= a_0 = \Sigma(D_r(x_r - 1) + E_r(x_r^{-1} - 1)) \quad . \quad (3) \\ y_2 - y_1 &= a_1 = \Sigma(D_r(x_r - 1)x_r + E_r(x_r^{-1} - 1)x_r^{-1}) \\ &\cdots \\ y_{k+1} - y_k &= a_k = \Sigma(D_r(x_r - 1)x_r^k + E_r(x_r^{-1} - 1)x_r^{-k}) \\ &\cdots \\ y_{3n} - y_{3n-1} &= a_{3n-1} = \Sigma(D_r(x_r - 1)x_r^{(3n-1)} + E_r(x_r^{-1} - 1)x_r^{-(3n-1)}). \end{aligned}$$

The number of constants, D, E, in each equation of set (3) is $2n$, therefore we can eliminate them from any $2n+1$ of these $3n$ equations. Taking the first $2n+1$, the result of the elimination may be expressed in the form—

$$\begin{vmatrix} a_0, & 1, & 1, & \ldots, & 1, & 1, \\ a_1, & x_1, & x_1^{-1}, & \ldots, & x_r, & x_r^{-1}, \\ a_2, & x_1^2, & x_1^{-2}, & \ldots, & x_r^2, & x_r^{-2}, \\ \cdots & & & & & \\ a_{2n}, & x_1^{2n}, & x_1^{-2n}, & \ldots, & x_r^{2n}, & x_r^{-2n}, \end{vmatrix} = 0 \quad . \quad (4)$$

Now let

$$x_1, x_1^{-1}, \ldots x_r, x_r^{-1}, \ldots x_n, x_{-n}^{-1},$$

be the roots of the equation

$$\lambda^{2n} + p_1\lambda^{2n-1} + p_2\lambda^{2n-2} + \ldots + p_2\lambda^2 + p_1\lambda + 1 = 0 \quad . \quad (5)$$

in which coefficients equidistant from the beginning and end are equal, since for all values of r, x_r, and x_r^{-1}, are roots. If we substitute the roots successively for λ in this equation, the result

must be identically zero. Hence, if we multiply the rows of the determinant (4) in order by the coefficients 1, p_1, p_2, etc., and add the elements of the rows so multiplied to the elements in the first row, all the terms in the first row except the first will vanish. The determinant then becomes

$$(a_0 + a_1p_1 + a_2p_2 + a_3p_3 + \ldots + a_{2n-1}p_1 + a_{2n})\mathrm{D} = 0,$$

where D denotes the determinant formed by suppressing the first row and column in (4). But D is not zero, hence

$$a_0 + a_1p_1 + a_2p_2 + a_3p_3 + \ldots + a_{2n-1}p_1 + a_{2n} = 0 \quad . \quad (6)$$

If we next take the $2n+1$ equations involving $a_1, a_2, \ldots, a_{2n+1}$, it is easily seen that after dividing the respective columns in the elimination determinant in order by $x_1, x_1^{-1}, \ldots, x_k, x_k^{-1}$, etc., and replacing a_1, a_2, a_3, etc., in order by a_0, a_1, a_2, etc., we again obtain (4). Hence

$$a_1 + a_2p_1 + a_3p_2 + \ldots + a_{2n}p_1 + a_{2n+1} = 0 \quad . \quad (6_1)$$

Proceeding in this manner, taking all the sets of consecutive $(2n+1)$ equations, we obtain from them equations of the same type as (6). Collecting terms with like coefficients, we can write (5) and the equations of form (6) as follows:—

$$\begin{aligned}
&(\lambda^n + \lambda^{-n}) + p_1(\lambda^{n-1} + \lambda^{-(n-1)}) + \ldots + p_{n-1}(\lambda + \lambda^{-1}) + p_n = 0 \quad (7)\\
&(a_0 + a_{2n}) + p_1(a_1 + a_{2n-1}) + \ldots + p_{n-1}(a_{n-1} + a_{n+1}) + p_n a_n = 0\\
&(a_1 + a_{2n+1}) + p_1(a_2 + a_{2n}) + \ldots + p_{n-1}(a_n + a_{n+2}) + p_n a_{n+1} = 0\\
&\qquad\qquad\qquad

Now, in determinant (8), if we call for the moment any element in the first column c_1, and the corresponding elements in the second, third, . . ., fifth columns, c_2, c_3, . . ., c_5, we can replace the first column by

$$c_1 + nc_3 + \frac{n^{(2)}}{\lfloor 2}c_5 + \ldots,$$

the second by

$$c_2 + (n-1)c_4 + \frac{(n-1)^{(2)}}{\lfloor 2}c_6 + \ldots,$$

and so on, the last column alone being unaltered. When this is done the determinant becomes

$$\begin{vmatrix} z^n, & z^{n-1}, & z^{n-2}, & \ldots z^2, & z, & 1 \\ \mathrm{E}^n a_n, & \mathrm{E}^{n-1}a_{n-1}, & \mathrm{E}^{n-2}a_n, & \ldots \mathrm{E}^2 a_n, & \mathrm{E}a_n, & a_n \\ \mathrm{E}^n a_{n+1}, & \mathrm{E}^{n-1}a_{n+1}, & \mathrm{E}^{n-2}a_{n+1}, & \ldots \mathrm{E}^2 a_{n+1}, & \mathrm{E}a_{n+1}, & a_{n+1} \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ \mathrm{E}^n a_{2n-1}, & \mathrm{E}^{n-1}a_{2n-1}, & \mathrm{E}^{n-2}a_{2n-1}, & \ldots \mathrm{E}^2 a_{2n-1}, & \mathrm{E}a_{2n-1}, & a_{2n-1} \end{vmatrix} = 0 \quad (11)$$

and this is the most convenient form for the equation in z.

The following table indicates how the elements in the determinant are found from the tabulated values of y :—

$$a_r = y_{r+1} - y_r,$$

$$\begin{array}{llllll} y_0 & a_0 & & & & \\ y_1 & a_1 & a_0 + a_2 = \mathrm{E}a_1 & & & \\ y_2 & a_2 & a_1 + a_3 = \mathrm{E}a_2 & \mathrm{E}a_1 + \mathrm{E}a_3 = \mathrm{E}^2 a_2 & & \\ y_3 & a_3 & a_2 + a_4 = \mathrm{E}a_3 & \mathrm{E}a_2 + \mathrm{E}a_4 = \mathrm{E}^2 a_3 & \mathrm{E}^2 a_2 + \mathrm{E}^2 a_4 = \mathrm{E}^3 a_3 & (12) \\ y_4 & a_4 & a_3 + a_5 = \mathrm{E}a_4 & \mathrm{E}a_3 + \mathrm{E}a_5 = \mathrm{E}^2 a_4 & & \\ y_5 & a_5 & a_4 + a_6 = \mathrm{E}a_5 & & & \\ y_6 & a_6 & & & & \\ y_7 & & & & & \end{array}$$

By obvious transformations (11) can be reduced to the form

$$\begin{vmatrix} z^n, & z^{n-1}, & z^{n-2}, & \ldots z^2 & z, & 1 \\ b_{0n}, & b_{0(n-1)}, & b_{0(n-2)}, & \ldots b_{02}, & b_0, & 1 \\ b_{1n}, & b_{1(n-1)}, & b_{1(n-2}, & \ldots b_1, & 1, & 0 \\ b_{2n}, & b_{2(n-1)}, & b_{2(n-2)}, & \ldots 1, & 0, & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ b_{(n-1)n}, & 1, & 0, & \ldots 0, & 0, & 0 \end{vmatrix} = 0 \quad (13)$$

and the labour of solution is thereby greatly diminished. The coefficient of z^n is now unity, and that of z^{n-1} is $b_{(n-1)n}$.

The solution obtained by using any set of equidistant values of y is, of course, not a unique solution since the angles θ_1, θ_2, etc., are determined from their cosines.

The proper angle to be chosen can only be settled by effecting

a solution with a different period of analysis. The advance of the argument must be proportional to the length of the period of analysis adopted.

The Coefficients.

3. The quantities x_r or θ_r having been determined, we must now find the coefficients B_r, C_r or D_r, E_r, the latter pair being related to the former by the equations

$$D_r + E_r = B_r, \quad i(D_r - E_r) = C_r. \qquad . \qquad . \quad (14)$$

Returning to equations (3), we take the first $2n$ equations to determine to $2n$ unknown quantities D_r, E_r ($r = 1, 2, \ldots n$).

If we solve for $D_1(x_1 - 1)$ we obtain

$$D_1(x_1 - 1) = \frac{F_1}{G_1} \qquad . \qquad . \qquad . \qquad . \quad (15)$$

where F_1 denotes

$$\begin{vmatrix} a_0, & 1, & 1, & 1, & \ldots, 1 \\ a_1, & x_1^{-1}, & x_2, & x_2^{-1}, & \ldots, x_n^{-1} \\ a_2, & x_1^{-2}, & x_2^{2}, & x_2^{-2}, & \ldots, x_n^{-2} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{2n-1}, & x_1^{-(2n-1)}, & x_2^{(2n-1)}, & x_2^{-(2n-1)}, & \ldots, x_n^{-(2n-1)} \end{vmatrix} \qquad (16)$$

and G_1 denotes

$$\begin{vmatrix} 1, & 1, & 1, & 1, & \ldots, 1 \\ x_1, & x_1^{-1}, & x_2, & x_2^{-1}, & \ldots, x_n^{-1} \\ x_1^2, & x_1^{-2}, & x_2^{2}, & x_2^{-2}, & \ldots, x_n^{-2} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ x_1^{2n-1}, & x_1^{-(2n-1)}, & x_2^{(2n-1)}, & x_2^{-(2n-1)}, & \ldots, x_n^{-(2n-1)} \end{vmatrix} \qquad (17)$$

To simplify these determinants let

$$f_1(\lambda) = \lambda^{2n-1} + q_1\lambda^{2n-2} + q_2\lambda^{2n-3} + \ldots + q_{2n-1} = 0. \qquad (18)$$

be the equation whose roots are

$$x_1^{-1}, x_2, x_2^{-1}, x_3, x_3^{-1}, \ldots x_n, x_n^{-1}.$$

Multiplying then the rows of each determinant in order from the bottom by

$$1, q_1, q_2, \ldots q_{2n-1},$$

and adding to the top row we find

$$F_1 = (a_{2n-1} + q_1a_{2n-2} + q_2a_{2n-3} \ldots + q_{2n-2}a_1 + q_{2n-1}a_0)\Delta$$

and

$$G_1 = (x_1^{2n-1} + q_1x_1^{2n-2} + q_2x_1^{2n-3} \ldots + q_{2n-2}x_1 + q_{2n-1})\Delta,$$

where Δ is the determinant formed by erasing the first row and column in either F_1 or G_1.

Hence

$$D_1(x_1 - 1) = \frac{a_{2n-1} + q_1a_{2n-2} + \ldots + q_{2n-2}a_1 + q_{2n-1}a_0}{x_1^{2n-1} + q_1x_1^{2n-2} + \ldots + q_{2n-2}x_1 + q_{2n-1}} \qquad (19)$$

Now

$$\begin{aligned} f_1(\lambda) &= (\lambda - x_1^{-1})(\lambda - x_2)(\lambda - x_2^{-1})(\lambda - x_3)(\lambda - x_3^{-1}) \ldots (\lambda - x_n)(\lambda - x_n^{-1}) \\ &= (\lambda - x_1^{-1})(\lambda^{2n-2} + r_1\lambda^{2n-3} + r_2\lambda^{2n-4} + \ldots + r_2\lambda^2 + r_1\lambda + 1) \\ &= \lambda^{2n-1} + \lambda^{2n-2}(r_1 - x_1^{-1}) + \lambda^{2n-3}(r_2 - r_1x_1^{-1}) + \ldots \lambda(1 - r_1x_1^{-1}) - x_1^{-1} \end{aligned}$$

where the r's denote coefficients of powers of λ.

Hence

$$\begin{aligned} q_1 &= r_1 - x_1^{-1} \\ q_2 &= r_2 - r_1x_1^{-1} \\ q_3 &= r_3 - r_2x_1^{-1} \\ q_{2n-1} &= -x_1^{-1}. \end{aligned}$$

Also

$$\begin{aligned} f_1(x_1) &= x_1^{2n-1} + q_1x_1^{2n-2} + \ldots + q_{2n-2}x_1 + q_{2n-1} \\ &= (x_1 - x_1^{-1})(x_1 - x_2)(x_1 - x_2^{-1})(x_1 - x_3)(x_1 - x_3^{-1}) \ldots (x_1 - x_n^{-1}). \end{aligned}$$

Substituting these values in (19), we obtain

$$D_1(x_1 - 1) = \frac{P_1 - Q_1x^{-1}}{(x_1 - x_1^{-1})(x_1 - x_2)(x_1 - x_2^{-1}) \ldots (x_1 - x_n)(x_1 - x_n^{-1})},$$

where

$$\left.\begin{aligned} P_1 &= a_{2n-1} + a_1 + r_1(a_{2n-2} + a_2) + r_2(a_{2n-3} + a_3) + \ldots r_na_n \\ Q_1 &= a_{2n-2} + a_0 + r_1(a_{2n-3} + a_1) + r_2(a_{2n-4} + a_2) + \ldots r_na_{n-1} \end{aligned}\right\} \quad (20)$$

We shall presently transform these expressions into another shape more suitable for calculation.

Considering the denominator, we see that we have $(n-1)$ pairs of factors such as

$$\begin{aligned} (x_1 - x_2)(x_1 - x_2^{-1}) &= x_1(x_1 - x_2)(1 - x_1^{-1}x_2^{-1}) \\ &= x_1(x_1 + x_1^{-1} - x_2 - x_2^{-1}) \\ &= 2x_1(\cos\theta_1 - \cos\theta_2). \end{aligned}$$

Hence

$$D_1(x_1 - 1) = \frac{P_1 - Q_1x_1^{-1}}{2^{n-1}x_1^{n-1}(x_1 - x_1^{-1}) \prod_{r=2}^{r=n}(\cos\theta_1 - \cos\theta_r)} \quad (21)$$

If we now solve for $E_1(x_1^{-1} - 1)$ we obtain a similar expression, P_1 and Q_1 being unaltered, and x_1 and x_1^{-1} being interchanged. Therefore

$$E_1(x_1^{-1} - 1) = \frac{P_1 - Q_1x_1}{2^{n-1}x_1^{-(n-1)}(x_1^{-1} - x_1) \prod_{r=2}^{r=n}(\cos\theta_1 - \cos\theta_r)},$$

or

$$E_1(x_1 - 1) = \frac{x_1(P_1 - Q_1x_1)}{2^{n-1}x_1^{-(n-1)}(x_1 - x_1^{-1}) \prod_{r=2}^{r=n}(\cos\theta_1 - \cos\theta_r)} \quad (22)$$

Adding (21) and (22), and taking account of (14),

$$B_1(x_1 - 1) = \frac{P_1 x_1^{-(n-1)} - Q_1 x_1^{-n} + P_1 x_1^{n} - Q_1 x_1^{n+1}}{2^{n-1}(x_1 - x_1^{-1}) \prod_{r=2}^{r=n} (\cos\theta_1 - \cos\theta_r)},$$

or

$$B_1(x_1^{\frac{1}{2}} - x_1^{-\frac{1}{2}}) = \frac{P_1(x_1^{(n-\frac{1}{2})} + x_1^{-(n-\frac{1}{2})}) - Q_1(x_1^{(n+\frac{1}{2})} + x_1^{-(n+\frac{1}{2})})}{2^{n-1}(x_1 - x_1^{-1}) \prod_{r=2}^{r=n} (\cos\theta_1 - \cos\theta_r)}.$$

Hence

$$B_1 = \frac{Q_1 \cos(n+\tfrac{1}{2})\theta_1 - P_1 \cos(n-\tfrac{1}{2})\theta_1}{2^n \sin\tfrac{1}{2}\theta_1 . \sin\theta_1 . \prod_{r=2}^{r=n} (\cos\theta_1 - \cos\theta_r)} \quad . \quad (23)$$

Again, subtracting (22) from (21),

$$C_1(x_1 - 1) = i\frac{P_1 x_1^{-(n-1)} - Q_1 x_1^{-n} - P_1 x_1^{n} + Q_1 x_1^{n+1}}{2^{n-1}(x_1 - x_1^{-1}) \prod_{r=2}^{r=n} (\cos\theta_1 - \cos\theta_r)},$$

or

$$C_1(x_1^{\frac{1}{2}} - x_1^{-\frac{1}{2}}) = i\frac{Q_1(x_1^{(n+\frac{1}{2})} - x_1^{-(n+\frac{1}{2})}) - P_1(x_1^{(n-\frac{1}{2})} - x_1^{-(n-\frac{1}{2})})}{2^{n-1}(x_1 - x_1^{-1}) \prod_{r=2}^{r=n} (\cos\theta_1 - \cos\theta_r)}.$$

Hence

$$C_1 = \frac{Q_1 \sin(n+\tfrac{1}{2})\theta_1 - P_1 \sin(n-\tfrac{1}{2})\theta_1}{2^n \sin\tfrac{1}{2}\theta_1 . \sin\theta_1 . \prod_{r=2}^{r=n} (\cos\theta_1 - \cos\theta_r)} \quad . \quad (24)$$

If we write

$$B_1 \cos\theta_1 t + C_1 \sin\theta_1 t = R_1 \sin(\theta_1 t + \alpha_1),$$
$$B_1 = R_1 \sin\alpha_1, \qquad C_1 = R_1 \cos\alpha_1,$$

and

$$\tan\alpha_1 = \frac{Q_1 \cos(n+\tfrac{1}{2})\theta_1 - P_1 \cos(n-\tfrac{1}{2})\theta_1}{Q_1 \sin(n+\tfrac{1}{2})\theta_1 - P_1 \sin(n-\tfrac{1}{2})\theta_1},$$

$$R_1 = \frac{(P_1^2 + Q_1^2 - 2P_1Q_1\cos\theta_1)^{\frac{1}{2}}}{2^n \sin\tfrac{1}{2}\theta_1 . \sin\theta_1 \prod_{r=2}^{r=n} (\cos\theta_1 - \cos\theta_r)}.$$

Returning to the expressions for P_1 and Q_1 (20), the r's are the coefficients in the expanded form of

$$(\lambda - x_2)(\lambda - x_2^{-1})(\lambda - x_3)(\lambda - x_3^{-1}) \ . \ . \ . \ (\lambda - x_n)(\lambda - x_n^{-1}) = 0.$$

Suppose that this becomes, on substituting z for $\lambda + \lambda^{-1}$,

$$z^{n-1} + r'_1 z^{n-2} + r'_2 z^{n-3} + \ . \ . \ . \ + r'_{n-1} = 0.$$

This equation is derived from the fundamental equation (11) by removing the factor $z - 2\cos\theta_1$. Hence if we divide the fundamental equation by $z - 2\cos\theta_1$, we find the r's.

Now, it is proved in the Appendix that if the equation in λ is

transformed into the equation in z, the coefficients (r_1, r_2, r_3, . . ., etc.), and (r'_1, r'_2, r'_3, . . ., etc.) are so related that

$$\begin{aligned} P_1 &= a_{2n-1} + a_1 + r_1(a_{2n-2} + a_2) + \quad \ldots + r_{n-1}a_n \\ &= E^{n-1}a_n + r'_1E^{n-2}a_n + r'_2E^{n-3}a_n \quad \ldots + r'_{n-1}a_n \,. \quad (25) \end{aligned}$$

and

$$\begin{aligned} Q_1 &= a_{2n-2} + a_2 + r_1(a_{2n-3} + a_3) \quad \ldots + r_{n-1}a_{n-1} \\ &= E^{n-1}a_{n-1} + r'_1E^{n-2}a_{n-1} + r'_2E^{n-3}a_{n-1} \ldots + r'_{n-1}a_{n-1} \quad (25') \end{aligned}$$

Thus when r'_1, r'_2 . . . have been found, P_1 and Q_1 are easily calculated.

The Number of Periods.

4. We have supposed the number of different periods to be known, but when this is not the case, they can be found by examination of a series of determinants In (7) we established a set of relations between any $(2n+1)$ consecutive a's, and by means of (9) and (10) these relations may be transformed into the shape

$$E^n a_r + p'_1E^{n-1}a_r + p'_2E^{n-2}a_r + \ldots + p'_{n-1}Ea_r + p'_n a_r = 0, \quad (26)$$

where p'_1, p'_2, . . ., p'_{n-1}, p'_n are nothing but the coefficients in the fundamental equation (11). (See Appendix.)

Now, if r is greater than $2n-1$,—in other words, if the number of observed values of 7 is greater than $3n+1$,—we can eliminate p'_1, p'_2, . . ., p'_n between any $n+1$ equations of the type (25), and obtain a relation between the coefficients which must be identically zero. The relation can be written in the form of a determinant of order $n+1$, and in fact differs only from (11) in the elements of the first row being replaced by

$$E^n a_r,\ E^{n-1}a_r,\ E^{n-2}a_r,\ \ldots,\ Ea_r,\ a_r.$$

If we now form in succession the determinants

$$\begin{vmatrix} Ea_1, & a_1 \\ Ea_r, & a_r \end{vmatrix} \quad \begin{vmatrix} E^2a_2, & Ea_2, & a_2 \\ E^2a_3, & Ea_3, & a_3 \\ E^2a_r, & Ea_r, & a_r \end{vmatrix} \quad \begin{vmatrix} E^3a_3, & E^2a_3, & Ea_3, & a_3 \\ E^3a_4, & E^2a_4, & Ea_4, & a_4 \\ E^3a_5, & E^2a_5, & Ea_5, & a_5 \\ E^3a_r, & E^2a_r, & Ea_r, & a_r \end{vmatrix},$$

and so on, where in each case r is taken to be greater than any of the preceding numerical suffixes, then the order of the determinant of lowest order which is first identically zero, is one greater than the number of independent periods.

It is now obvious that in forming the equation for the periods we may make use of any n equations of type (26) and

$$z^n + p'_1z^{n-1} + p'_2z^{n-2} + \ldots + p'_{n-1}z + p'_n = 0,$$

between which p'_1, p'_2, . . ., p'_n are to be eliminated and a determinantal equation of the form (11) obtained.

Moreover, any equation (26) may be replaced by any linear combination of equations of the same type; for example, the mean of several may be used. This plan would naturally be adopted when a large number of observed values of y are available, in order to eliminate accidental errors.

The criterion established for determining the number of periods assumes that the data are of an infinite degree of exactness. Since this is not the case, we must modify the statement. To fix ideas, consider the case of two periods. If two periods only exist, the coefficients in the fundamental equation are p'_1, p'_2, and are found by solving

$$\mathrm{E}^2a_2 + p'_1\mathrm{E}a_2 + p'_2a_2 = 0$$
$$\mathrm{E}^2a_3 + p'_1\mathrm{E}a_3 + p'_2a_3 = 0.$$

Hence

$$\frac{1}{\begin{vmatrix} \mathrm{E}a_2, & a_2 \\ \mathrm{E}a_3, & a_3 \end{vmatrix}} = \frac{p'_1}{\begin{vmatrix} a_2, & \mathrm{E}^2a_2 \\ a_3, & \mathrm{E}^2a_3 \end{vmatrix}} = \frac{p'_2}{\begin{vmatrix} \mathrm{E}^2a_2, & \mathrm{E}a_2 \\ \mathrm{E}^2a_3, & \mathrm{E}a_3 \end{vmatrix}}$$

Now these values of p'_1, p'_2, should make $\mathrm{E}^2a_r + p'_1\mathrm{E}a_r + p'_2a_r$ vanish.

On substituting, this expression becomes

$$\begin{vmatrix} \mathrm{E}^2a_2, & \mathrm{E}a_2, & a_2 \\ \mathrm{E}^2a_3, & \mathrm{E}a_3, & a_3 \\ \mathrm{E}^2a_r, & \mathrm{E}a_r, & a_r \end{vmatrix} \div \begin{vmatrix} \mathrm{E}a_2, & a_2 \\ \mathrm{E}a_3, & a_3 \end{vmatrix}$$

and if we find the value of this quotient to be within the permissible limits of error, when we take account of the limited accuracy of the data, then we have strong ground for concluding that all the conditions will be satisfied by assuming only two periods to be involved. This conclusion becomes practically a certainty if we find the value of the expression to be within the limits of error for all different values of r which are available for examination.

We can form an estimate of the permissible limits of error by the following considerations. If we suppose that the values of y are correct to the nearest final integer tabulated, their possible error is at the most half a unit in the last place. Therefore the possible errors in the a's are one unit, in $\mathrm{E}a$ two units, and in $\mathrm{E}^n a$ 2^n units of the last tabulated place.

Now for possible solutions the roots of the equation in z cannot exceed 2. Hence in equation (26) the maximum value of p' is $2n$, of p'_2, $\frac{n^{(2)}}{\lfloor 2}2^2$, . . ., of p'_r, $\frac{n^{(r)}}{\lfloor r}2^r$. . . . Hence the greatest possible error in the left-hand side of (26), due to errors in the tabulated values, is

$$2^n + n \,.\, 2 \,.\, 2^{n-1} + \frac{n^{(2)}}{\lfloor 2} \,.\, 2^2 \,.\, 2^{n-2} + \ldots \frac{n^{(r)}}{\lfloor r}2^r \,.\, 2^{n-r} + \ldots 2^n = 4^n.$$

If we call the successive determinants formed as above, Δ_1, Δ_2, Δ_3, Δ_n, when the suffix denotes the order of the determinant, we have a practical rule that we may at the outset of any investigation take the number of independent periods to be n, where n is the smallest integer for which the inequality

$$\frac{\Delta_{n+1}}{\Delta'_n} < 4^n \quad . \quad . \quad . \quad . \quad (27)$$

is satisfied where Δ'_n denotes the determinant formed by erasing in Δ_{n+1} one of the rows and columns containing E^n.

Finally, we notice that in equation (26) r may have any value integral or fractional, and thereby gain a great degree of freedom in forming our equations.

5. When a large number of observations are available they may be combined with one another for the purpose of forming the coefficients which appear in the equation for the periods. Suppose we combine the observations for which

$$t = 0, \quad t_1, t_2, \quad . \; . \; ., t_{s-1},$$

then those for which

$$t = 1, \quad 1 + t_1, \quad 1 + t_2, \quad 1 + t_{s-1}$$

and in general those for which

$$t = k, \quad k + t_1, \quad k + t_2, \quad k + t_{s-1}$$

where k is a positive integer.

Then y_k is replaced by

$$\begin{aligned} y'_k &= y_k + y_{k+t_1} + y_{k+t_2} \; . \; . \; . \; + y_{k+t_{s-1}} \\ &= sA + \Sigma D_r(x_r^k(1 + x_r^{t_1} + x_r^{t_2} + \; . \; x_r^{t_{s-1}})) \\ &\qquad + \Sigma E_r(x_r^{-k}(1 + x_r^{-t_1} + x_r^{-t_2} + \; . \; x_r^{-t_{s-1}})) \\ &= sA + \Sigma D_r \xi x_r^k + \Sigma E_r \xi' x_r^{-k}, \end{aligned}$$

where ξ and ξ' are the same for all values of k. Then a_k is replaced by a'_k where

$$a'_k = y'_{k+1} - y'_k = \Sigma(D_r \xi(x - 1)x_r^k + E_r \xi'(x^{-1} - 1)x_r^k),$$

and in forming the equations for the periods we eliminate $D_r\xi(x-1)$ and $E_r\xi'(x^{-1}-1)$ instead of simply $D_r(x-1)$ and $E_r(x^{-1}-1)$.

Therefore the equation for the periods is of the same form as before, a_k being replaced by a'_k defined as above.

The expressions obtained for the constants will require to be modified. The following particular case will be useful later. Suppose the period of analysis to be divided into n equal sub-periods, and we combine the values of y corresponding to

$$t = k, \quad k + \frac{1}{m}, \quad k + \frac{2}{m} + \; . \; . \; . \; k + \frac{s-1}{m}$$

for all the different values of k. Then it can be easily proved

that the expressions for B_1 and C_1 obtained in section 3, must be replaced by

$$B_1 = \frac{\sin\frac{\theta_1}{2m}\left(Q_1\cos\left(n+\frac{1}{2}+\frac{s-1}{2m}\right)\theta_1 - P_1\cos\left(n-\frac{1}{2}+\frac{s-1}{2m}\right)\theta_1\right)}{2^n\sin\theta_1\,.\,\sin\frac{\theta_1}{2}\,.\,\sin\frac{s\theta_1}{2m}\,.\,\prod_{r=2}^{r=n}(\cos\theta_1-\cos\theta_r)} \quad (28)$$

$$C_1 = \frac{\sin\frac{\theta_1}{2m}\left(Q_1\sin\left(n+\frac{1}{2}+\frac{s-1}{2m}\right)\theta_1 - P_1\sin\left(n-\frac{1}{2}+\frac{s-1}{2m}\right)\theta_1\right)}{2^n\sin\theta_1\,.\,\sin\frac{\theta_1}{2}\,.\,\sin\frac{s\theta_1}{2m}\,.\,\prod_{r=2}^{r=n}(\cos\theta_1-\cos\theta_r)} \quad (29)$$

6. The above analysis may be applied without further difficulty to the case in which the function is expressible partly algebraically and partly trigonometrically in terms of t.

For example, if the expression for y is

$$y = A + Bt + \Sigma(C_r\cos\theta_r t + D_r\sin\theta_r t),$$

we eliminate A by taking the first differences and B by taking the second differences of y. The equation for determining θ_r will be of the same form as before, the only change being the altered signification of the a's, which now denote second differences. There will also be a small change in the expressions which give the coefficients.

7. To illustrate the foregoing theory we shall determine the combination of periodic functions which gives the values of y shown in the following table. We first form the values of a, Ea, and E^2a defined by equations (3) and (9).

t.	y.	a.	Ea.	E^2a.
0	2·6840	+ 1·5480		
1	4·2320	+ ·6036	+ ·9980	
2	4·8356	− ·5500	− ·8160	− 1·2036
3	4·2856	− 1·4196	− 2·2016	− 3·4196
4	2·8660	− 1·6516	− 2·6036	
5	1·2144	− 1·1840		
6	0·0304			

The equation for the arguments θ_1 and θ_2 is

$$\begin{vmatrix} z^2 & z & 1 \\ 1·2036 & ·8160 & ·5500 \\ 3·4196 & 2·2016 & 1·4196 \end{vmatrix} = 0.$$

Writing the logarithms of numbers within square brackets, this becomes

$$\begin{vmatrix} z^2 & z & 1 \\ [\cdot 080482] & [\bar{1}\cdot 911690] & [\bar{1}\cdot 740363] \\ [\cdot 533975] & [\cdot 342738] & [\cdot 152166] \end{vmatrix} = 0.$$

Hence

$$\begin{vmatrix} z^2 & z & 1 \\ [\cdot 340119] & [\cdot 171327] & 1 \\ [\cdot 381809] & [\cdot 190572] & 1 \end{vmatrix} = 0.$$

or

$$\begin{vmatrix} z^2 & z & 1 \\ 2\cdot 18836 & 1\cdot 48363 & 1 \\ 2\cdot 40884 & 1\cdot 55086 & 1 \end{vmatrix} = 0.$$

A further reduction leads to

$$\begin{vmatrix} z^2 & z & 1 \\ 2\cdot 18836 & 1\cdot 48363 & 1 \\ 3\cdot 27949 & 1 & 0 \end{vmatrix} = 0.$$

or

$$z^2 - 3\cdot 27949z + 2\cdot 67720 = 0.$$

The roots of this equation are

$$1\cdot 74733 \text{ and } 1\cdot 53216.$$

Hence

$$\cos\theta_1 = \cdot 87367, \qquad \cos\theta_2 = \cdot 76608,$$

and

$$\theta_1 = 29^\circ\cdot 11, \qquad \theta_2 = 39^\circ\cdot 99.$$

Next, to calculate the coefficients B, C. Here $n = 2$, and the coefficients corresponding to θ_1 are

$$B_1 = R_1 \sin\alpha_1 = \frac{Q_1 \cos\frac{5}{2}\theta_1 - P_1 \cos\frac{3}{2}\theta_1}{4\sin\frac{1}{2}\theta_1 \cdot \sin\theta_1 \cdot (\cos\theta_1 - \cos\theta_2)},$$

$$C_1 = R_1 \cos\alpha_1 = \frac{Q_1 \sin\frac{5}{2}\theta_1 - P_1 \sin\frac{3}{2}\theta_1}{4\sin\frac{1}{2}\theta_1 \cdot \sin\theta_1 (\cos\theta_1 - \cos\theta_2)},$$

$$P_1 = Ea_2 - 2\cos\theta_2 \cdot a_2, \quad Q_1 = Ea_1 - 2\cos\theta_2 a_1.$$

Logarithms being denoted by square brackets, we find—

$$P_1 = +0\cdot 0269 = +[\bar{2}\cdot 4298], \quad Q_1 = +0\cdot 0731 = +[\bar{2}\cdot 8639],$$

$$4\sin\tfrac{1}{2}\theta_1 \cdot \sin\theta_1(\cos\theta_1 - \cos\theta_2) = +[\bar{2}\cdot 7200].$$

$$Q_1 \cos\frac{5\theta_1}{2} = \cdot 02165, \quad P_1 \cos\tfrac{3}{2}\theta_1 = \cdot 01946.$$

$$\text{Difference} = 0\cdot 00219 = [\bar{3}\cdot 3404].$$

Hence

$$B_1 = [\bar{3}\cdot3404] - [\bar{2}\cdot7200] = [\bar{2}\cdot6204] = \cdot04173.$$

$$Q_1 \sin \frac{5\theta_1}{2} = \cdot06980, \quad P_1 \sin \tfrac{3}{2}\theta_1 = \cdot01857.$$

$$\text{Difference} = 0\cdot05123 = [\bar{2}\cdot7096].$$

Hence

$$C_1 = [\bar{2}\cdot7096] - [\bar{2}\cdot7200] = [\bar{1}\cdot9896] = \cdot9765;$$
$$\tan \alpha_1 = B_1/C_1 = [\bar{2}\cdot6308], \quad \therefore \alpha = \tan 2^\circ\cdot45;$$
$$\sin \alpha_1 = [\bar{2}\cdot6304], \quad \therefore R_1 = [\bar{1}\cdot9900] = \cdot9772.$$

In a similar manner we find for B_2 and C_2,

$$P_2 = +0\cdot1452 = +[\bar{1}\cdot1620], \quad Q_2 = -\cdot0566 = -[\bar{2}\cdot7528],$$
$$4 \sin \tfrac{1}{2}\theta_2 \,.\, \sin \theta_2(\cos \theta_2 - \cos \theta_1) = -[\bar{2}\cdot9761],$$

$$Q_2 \cos \frac{5\theta_2}{2} = \cdot00983, \quad P_2 \cos \tfrac{3}{2}\theta_2 = +\cdot07261.$$

$$\text{Difference} = -\cdot06278 = -[\bar{2}\cdot7979].$$

Hence

$$B_2 = [\bar{2}\cdot7979] - [\bar{2}\cdot9761] = [\bar{1}\cdot8218] = \cdot6634,$$
$$Q_2 \sin \tfrac{5}{2}\theta_2 = -\cdot05574, \quad P_2 \sin \tfrac{3}{2}\theta_2 = +\cdot12575.$$
$$\text{Difference} = -\cdot1815 = -[\bar{1}\cdot2589].$$

Hence

$$C_2 = [\bar{1}\cdot2589] - [\bar{2}\cdot9761] = [\cdot2828] = 1\cdot918;$$
$$\tan \alpha_2 = B_2/C_2 = [\bar{1}\cdot5390], \quad \therefore \alpha_2 = 19^\circ\cdot08,$$
$$\sin \alpha_2 = [\bar{1}\cdot5144], \quad \therefore R_2 = [\cdot3074] = 2\cdot030.$$

To find A, take the value of y corresponding to $t = 0$.

Then $$2\cdot6840 = A + B_1 + C_1 = A + \cdot7051.$$

Hence $$A = 1\cdot9789.$$

Hence

$$y = 1\cdot9789 + \cdot9772 \sin(29^\circ\cdot11t + 2^\circ\cdot45) + 2\cdot030 \sin(40^\circ t + 19^\circ\cdot08).$$

As a matter of fact, the expression from which the values of y were calculated was

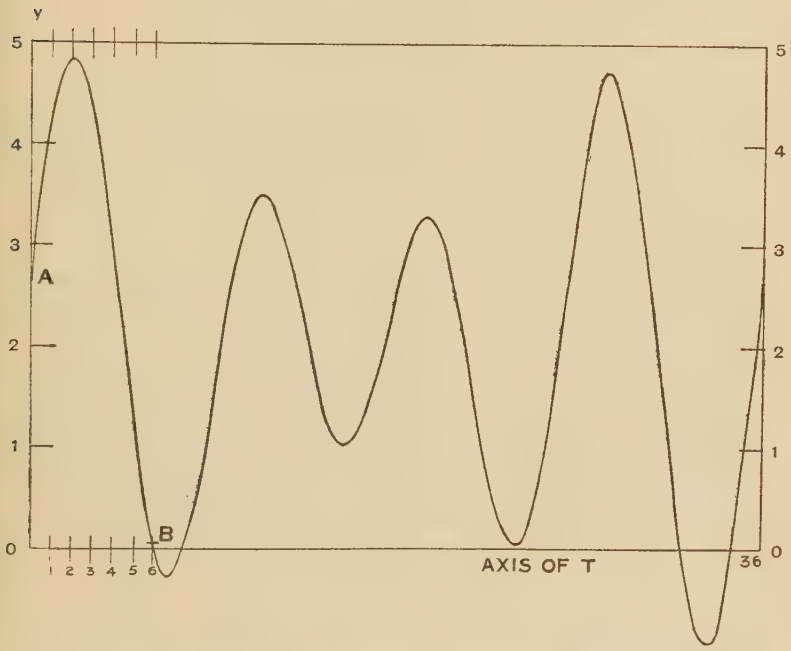
$$y = 2 + \sin(30^\circ t) + 2 \sin(40^\circ t + 20^\circ).$$

This function repeats itself in a time $t = 36$, while the data we used extended only from $t = 0$ to $t = 6$—that is, over one-sixth of the complete period.

8. The analysis has therefore determined the character of the whole function with a very satisfactory degree of accuracy considering the small fraction of the whole period which was examined. A still closer agreement could doubtless be obtained if more values of y were given by varying the constants and equating the resulting expression to the residuals. On the other hand, it must be noticed

that the values of y are given with a considerable degree of accuracy, and that a change of 1 per cent. in the values would considerably alter the character of the result.

Now many of the data in astronomical questions, such as sun-spot numbers, or the magnitudes of variable stars, can hardly be regarded as possessing an accuracy of even one per cent., and it may be doubted whether it is worth the trouble to apply to them in a rigorous manner the method of analysis developed in this paper.



$y = 2 + \sin(30° t) + 2 \sin(40° t + 20°)$. A B is the part analysed.

On referring to (25) we see that if the function contained only one period, the a's would satisfy the equation

$$Ea_r + p'_1 a_r = 0,$$

where in this case $p'_1 = -2 \cos \theta_1$, so that

$$\cos \theta_1 = \frac{Ea_r}{2a_r}.$$

Now in general $\frac{Ea_r}{2a_r}$ will not be constant, but if we find it maintains a fairly constant value, we shall be justified in assuming that there is a periodicity of predominant amplitude in the function whose value is determined by the average value of $\frac{Ea_r}{2a_r}$.

Suppose now that we have in this manner discovered a dominant periodicity, equation (25) suggests a mode of searching for another. Assuming two periodicities to exist, the equation connecting the a's becomes

$$E^2a_r + p'_1 Ea_r + p'_2 a_r = 0,$$

or writing p'_1, p'_2 in terms of the cosines of the arguments θ_1 and θ_2—

$$E^2a_r - 2(\cos\theta_1 + \cos\theta_2)Ea_r + 4\cos\theta_1\cos\theta_2 a_r = 0.$$

Hence

$$\cos\theta_2 = \frac{E^2a_r - 2\cos\theta_1 Ea_r}{2(Ea_r - 2\cos\theta_1 a_r)}.$$

Now, if we choose our unit of time so that θ_1 is 90° or nearly so, $\cos\theta_1$ will be very small and the value of $\cos\theta_2$ will be $E^2a_r/2(Ea_r)$.

If this quantity remained nearly constant, we should feel justified in assuming that it indicates the existence of another component periodicity.

9. As a further example of the methods developed in this paper, we shall examine Newcomb's "Minor Residuals" in the fluctuations in the Moon's Mean Motion. (*Mon. Not.*, vol. lxix. p. 166.)

From 1848·5 to 1908·2 annual values are given, with the exception of the value for 1877·5. In the following work the value −1·9 has been assigned for this year, interpolating between adjoining values. In a preliminary examination the residuals were grouped in periods of eight years, and the sum of the first five in each group taken. Values of a'_k (§5) were thus obtained—

a'_0.	a'_1.	a'_2.	a'_3.	a'_4.	a'_5.	a'_6.
+106	−49	−206	+15	−74	+100	+141

(The unit is 0″·1.)

Determinants for finding the number of periods were then formed and it was found that the value of Δ_3/Δ'_2 was 53. Now since five terms have been grouped together to form each y', the permissible limit for the value of this ratio is $5 \times 4^2 = 80$. Hence we may assume at the outset that the number of independent periods is two. On forming the quadratic for the periods and solving, we obtain

$$\cos\theta_1 = \cdot 640, \quad \cos\theta 2 = -\cdot 763.$$

Hence

$$\theta_1 = n \,.\, 360^\circ \pm 50^\circ\cdot 24, \quad \theta_2 = n \,.\, 360^\circ \pm 139^\circ\cdot 64,$$

n being any integer.

The advances of the arguments per annum are therefore

$$\theta_1/8 = n \,.\, 45^\circ \pm 6^\circ\cdot 28 \text{ and } \theta_2/8 = n \,.\, 45^\circ \pm 17^\circ\cdot 45.$$

The residuals were next grouped in periods of seven years, beginning with 1848·5, and the following table found:—

	y'.	a'.	Ea'.	E^2a'.
0	+ 68	+139		
1	+207	− 34	−105	
2	+173	−244	− 70	−434
3	− 71	− 36	−329	− 96
4	−107	− 85	− 26	−273
5	−192	+ 10	+ 56	
6	−182	+141		
7	− 41			

We have therefore three equations available from which the quadratic for the roots is to be formed—

$$\begin{aligned} 434 + 70p_1 + 244p_2 &= 0 \\ 96 + 329p_1 + 36p_2 &= 0 \\ 273 + 26p_1 + 85p_2 &= 0. \end{aligned}$$

It is desirable to make use of all of them, and the question arises how they should be continued.

Now the probable error of the numbers in the first column is twice that of those in the second, and the error of those in the second twice that of those in the third. It was therefore decided to continue them roughly in the least square manner, the unknowns being p_1 and p_2. The equations were multiplied in order by 3, 12, 1, and then by 7, 1, 2, numbers nearly proportional to the coefficients of p_1 and p_2 in the several equations.

The resulting quadratic for the period obtained in this way is

$$\begin{vmatrix} z^2 & z & 1 \\ 2727 & 4184 & 1249 \\ 3680 & 871 & 1914 \end{vmatrix} = 0.$$

The roots are 1·41755 and −1·32745; hence the values of $\cos\theta_1$ and $\cos\theta_2$ derived from these equations are

$$\cos\theta_1 = \cdot 70877, \quad \cos\theta_2 = -\cdot 66372.$$

The corresponding values of the angles are

$$\theta_1 = n.360^\circ \pm 44^\circ\cdot 86, \quad \theta_2 = n.360^\circ \pm 131^\circ\cdot 58.$$

Hence for the annual advances we find

$$\theta_1/7 = n.51^\circ\cdot 43 \pm 6^\circ\cdot 409, \quad \theta_2/7 = n.51^\circ\cdot 43 \pm 18^\circ\cdot 80.$$

Comparing these with the values previously obtained, it is clear that the only value of n satisfying both sets of equations is zero.

Hence we take with the seven-year period

$$\theta_1 = 44^\circ\cdot 86 \text{ and } \theta_2 = 131^\circ\cdot 58.$$

The first of these corresponds to a period of 56·19 years, and the second to one of 19·15 years.

The values obtained from the seven-year analysis are preferred

to those derived from the eight-year, because only about two-thirds of the available data were utilised in forming the equations in the latter case.

In determining the constants we use formulæ (28) and (29). These expressions involve Q and P where

$$P = Ea_2 - 2\cos\theta a_2, \quad Q = Ea_1 - 2\cos\theta a_1.$$

The values of B and C thus obtained determine the amplitude and phase at the beginning of the period of analysis. But if we change a_1 into a_2 and a_2 into a_3 and calculate the values of B and C, we shall obtain the amplitude and phase of the beginning of the next period.

If then we write $V_r = Ea_r - 2\cos\theta a_r$, we determine the values of B and C at the beginning of the r^{th} period of analysis by equations of the form—

$$B = K_1V_r + K_2V_{r+1}, \quad C = S_1V_r + S_2V_{r+1},$$

where K_1, K_2, S_1, S_2 retain the same values throughout.

We set out the calculation for the constants related to the angle θ_1—

$$n = 2, \quad m = 7, \quad s = 7.$$

$$B_1 = \frac{\sin\frac{\theta_1}{14}\left(V_r\cos\left(3-\frac{1}{14}\right)\theta_1 - V_{r+1}\cos\left(2-\frac{1}{14}\right)\theta_1\right)}{2\sin\theta_1(1-\cos\theta_1)(\cos\theta_1-\cos\theta_2)},$$

and C_1 is a similar expression with $\sin\left(3-\frac{1}{14}\right)\theta_1$ and $\sin\left(2-\frac{1}{14}\right)\theta_1$ substituted for the corresponding cosines.

$\cos\theta_1 = +\ \cdot70877,$	$\theta_1 = 44^\circ\cdot86.$
$\cos\theta_2 = -\ \cdot66372,$	$\theta_1/14 = 3^\circ\cdot204.$
$\cos\theta_1 - \cos\theta_2 = +1\cdot37249,$	$\left(2-\frac{1}{14}\right)\theta_1 = 86^\circ\cdot52.$
$1-\cos\theta_1 = \cdot29123,$	$\left(3-\frac{1}{14}\right)\theta_1 = 131^\circ\cdot38.$
$\sin\frac{\theta_1}{14} = [\bar{2}\cdot7473],$	$\cos\left(3-\frac{1}{14}\right)\theta_1 = -[\bar{1}\cdot8202]$
$2 = [\ \cdot3010],$	$[2\cdot9965]$
$\sin\theta_1 = [\bar{1}\cdot8484],$	$K_1 = -[\bar{2}\cdot8167]$
$1-\cos\theta_1 = [\bar{1}\cdot4639],$	$\sin\left(3-\frac{1}{14}\theta_1\right) = +[\bar{1}\cdot8752]$
$\cos\theta_1-\cos\theta_2 = [\ \cdot1375],$	$[\bar{2}\cdot9965]$
$\text{Product} = [\bar{1}\cdot7508],$	$S_1 = +[\bar{2}\cdot8717]$
$\text{Quotient} = [\bar{2}\cdot9965].$	
$-\cos\left(2-\frac{1}{14}\right)\theta_1 = -[\bar{2}\cdot7832],$	$-\sin\left(2-\frac{1}{14}\right)\theta_1 = -[\bar{1}\cdot9992]$
$[\bar{2}\cdot9965],$	$[\bar{2}\cdot9965]$
$K_2 = -[\bar{3}\cdot7797],$	$S_2 = -[\bar{2}\cdot9957]$

	Ea'.	a'.	$-2a'\cos\theta_2$.	V.	K_1V.	K_2V.	B.
1	−105	− 34	− 45	−150	+ 9·84	...	+12·21
2	− 70	−244	−324	−394	+25·84	+2·37	+28·11
3	−329	− 36	− 48	−377	+24·72	+2·27	+25·56
4	− 26	− 85	−113	−139	+ 9·11	+ ·84	+ 8·70
5	+ 56	+ 10	+ 13	+ 69	...	− ·42	

r.	S_1V.	S_2V.	C.	$\alpha_1+\overline{r-1}\theta_1$.	R_1.
1	−11·16	...	+27·85	23°·67	3″·04
2	−29·32	39·01	+ 8·01	74°·10	2″·92
3	−28·05	37·33	−14·29	119°·21	2″·93
4	−10·34	13·76	−17·18	153°·15	1″·92
5	...	− 6·83			

The two last columns contain respectively the phase and amplitude corresponding to the beginning of the first four periods of analysis. The change of phase between each period ought to be 44°·9, and the amplitudes should be equal. The average change of phase between the first and last is 43°·2, but the two middle phases are in advance of their proper values. Subtracting the proper multiples of 44°·86 and taking the mean, we obtain 25°·2 for the phase value in 1848·5.

Three of the values of R_1 are approximately equal, but the last value is notably smaller, about two-thirds. The only explanation that can be suggested is that it is connected in some way with errors in the small values − 26 and + 10 of Ea' and a' in periods 4 and 5. The eight-year analysis gave an amplitude nearly 3. We therefore reject 1″·92 and take 2″·96 for the value of the amplitude. Hence we obtain a term—

$$3''\cdot0\sin(6^\circ\cdot41(t-1848\cdot5)+25^\circ\cdot2)\quad .\quad .\quad .\quad (\alpha)$$

The determination of the phases and amplitudes corresponding to θ_2 proceeds in the same manner. We obtain finally—

r.	$\alpha_2+\overline{r-1}\theta_2$.	R_2.
1	186°·05	1″·15
2	311°·13	1″·08
3	82°·72	1″·08
4	230°·2	0″·60

Again, the value of R_2 corresponding to period 4 is about two-thirds of the others.

The concluded phase value is 185°·2, and the value of the term

$$1''\cdot11\sin(18^\circ\cdot8(t-1848\cdot5)+185^\circ\cdot2)\quad .\quad .\quad (\beta)$$

The value of the constant is found by subtracting the sum of the values of the two terms thus obtained. It is

$$-0''\cdot22\quad .\quad .\quad .\quad .\quad (\gamma)$$

In *Monthly Notices*, vol. lxxii. p. 27, F. E. Ross, after correcting Newcomb's places by

$$+0''\cdot50\sin\Omega\ (\Omega=\text{long. of Moon's node}),$$

obtains a correction to Newcomb's mean longitude—

$$-0''\cdot18+2''\cdot9\sin(350^\circ\cdot6+6^\circ\cdot316(t-1900\cdot0))$$
$$+0''\cdot8\sin(313^\circ\cdot6+15^\bullet\cdot65(t-1900\cdot0)).$$

These terms are of periods fifty-seven and twenty-three years respectively.

The first two terms correspond closely to the terms (α) and (γ).

In *Monthly Notices*, lxxiii. p. 702, Professor E. W. Brown corrects Newcomb's results by adding

$$-1''\cdot075\sin\Omega+0''\cdot060\cos\Omega.$$

Inserting the value of Ω and reducing to the epoch 1848·5, this becomes

$$-1''\cdot08\sin(19^\circ\cdot34t+182^\circ\cdot02).$$

This agrees fairly well with the term (β). The value of this correction is double of that applied by Ross, and the twenty-three-year period term may be regarded as an effort of his analysis to raise his coefficient to the proper amount.

Appendix.

If the equation

$$p_0(\lambda^n+\lambda^{-n})+p_1(\lambda^{n-1}+\lambda^{-(n-1)})+\ldots+p_r(\lambda^{n-r}+\lambda^{-(n-r)})+\ldots+p_n=0$$

is transformed into

$$q_0z^n+q_1z^{n-1}+q_2z^{n-2}+\quad\ldots\quad+q_rz^{n-r}+\quad\ldots\quad+q_n=0$$

by the substitution $z=\lambda+\lambda^{-1}$, *then*

$$p_0(a_0+a_{2n})+p_1(a_1+a_{2n-1})+\ldots+p_r(a_r+a_{2n-r})+\quad\ldots\quad+p_na_n$$

is transformed into

$$q_0\mathrm{E}^na_n+q_1\mathrm{E}^{n-1}a_n+q_2\mathrm{E}^{n-2}a_n+\ldots+q_r\mathrm{E}^{n-r}a_n+\quad\ldots\quad+q_na_n$$

where $a_0, a_1, a_2, \ldots a_{2n}$ *are any constants.*

Let $\lambda^n+\lambda^{-n}\equiv u_n$, $\lambda^{n-r}+\lambda^{n-r}\equiv u_{n-r}$, etc., then

$$z^n=(\lambda+\lambda^{-1})^n$$
$$=(\lambda^n+\lambda^{-n})+n(\lambda^{n-2}+\lambda^{-(n-2)})+\frac{n^{(2)}}{\lfloor 2}(\lambda^{n-4}+\lambda^{-(n-4)})+\ldots$$
$$=u_n+nu_{n-2}+\frac{n^{(2)}}{\lfloor 2}u_{n-4}+\ldots\frac{n^{(r)}}{\lfloor r}u_{n-2r}+\ldots$$

the last term being

$$\frac{n^{(\frac{1}{2}(n-1))}}{\lfloor\frac{1}{2}(n-1)}u_1,\quad\text{or}\quad\frac{n^{(\frac{1}{2}n)}}{\lfloor\frac{1}{2}n}$$

according as n is even or odd.

(Here $n^{(r)}\equiv n(n-1)(n-2)\ldots(n-r+1)$).

Hence

$$q_0z^n + q_1z^{n-1} + q_2z^{n-2} + q_3z^{n-3} \ldots + q_rz^{n-2} \ldots + q_n$$

$$\equiv q_0\left(u_n + nu_{n-2} + \frac{n^{(2)}}{\lfloor 2}u_{n-4} + \frac{n^{(3)}}{\lfloor 3}u_{n-6} + \ldots + \frac{n^{(r)}}{\lfloor r}u_{n-2r} + \ldots\right)$$

$$+ q_2\left(u_{n-2} + (n-2)u_{n-4} + \frac{(n-2)^{(2)}}{\lfloor 2}u_{n-6} + \ldots + \frac{(n-2)^{(r-1)}}{\lfloor r-1}u_{n-2r} + \ldots\right)$$

$$+ q_4\left(u_{n-4} + (n-4)u_{n-6} + \ldots + \frac{(n-4)^{(r-2)}}{\lfloor r-2}u_{n-2r} + \ldots\right)$$

$$+ \ldots$$

$$+ q_1\left(u_{n-1} + (n-1)u_{n-3} + \frac{(n-1)^{(2)}}{\lfloor 2}u_{n-5} + \frac{(n-1)^{(3}}{\lfloor 3}u_{n-7} + \ldots \frac{(n-1)^{(r)}}{\lfloor r}u_{n-2r-1} + \ldots\right)$$

$$+ q_3\left(u_{n-3} + (n-3)u_{n-5} + \frac{(n-3)^{(2)}}{\lfloor 2}u_{n-7} + \ldots \frac{(n-3)^{(r-1)}}{\lfloor r-1}u_{n-2r-1} + \ldots\right.$$

$$+ q_5\left(u_{n-5} + (n-5)u_{n-7} + \ldots \frac{(n-5)^{(r-2)}}{\lfloor r-2}u_{n-2r-1} + \ldots\right.$$

$$+ \ldots$$

But this is the same as

$$p_0u_n + p_1u_{n-1} + p_2u_{n-2} \ldots + p_ru_{n-r} + \ldots + p_n.$$

Therefore

$$p_0 = q_0$$

$$p_2 = nq_0 + q_2$$

$$p_4 = \frac{n^{(2)}}{\lfloor 2}q_0 + (n-2)q_2 + q_4$$

$$\ldots$$

$$p_{2s} = \frac{n^{(s)}}{\lfloor s}q_0 + \frac{(n-2)^{(s-1)}}{\lfloor s-1}q_2 + \ldots + (n-2s+2)q_{2s-2} + q_{2s}$$

$$p_1 = q_1$$

$$p_3 = (n-1)q_1 + q_3$$

$$p_5 = \frac{(n-1)^{(2)}}{\lfloor 2}q_1 + (n-3)q_3 + q_5$$

$$\ldots$$

$$p_{2s+1} = \frac{(n-1)^{(s)}}{\lfloor s}q_1 + \frac{(n-3)^{(s-1)}}{\lfloor s-1}q_3 + \ldots + (n-2s+1)q_{2s-1} + q_{2s+1}$$

Substituting these values in the expression

$$p_0(a_0+a_{2n})+p_1(a_1+a_{2n-1})+p_2(a_2+a_{2n-2})+\ldots\ p_r(a_r+a_{2n-r})+p_n a_n$$

it becomes

$$\begin{aligned}
&(a_0+a_{2n})q_0\\
&+(a_2+a_{2n-2})(nq_0+q_2)\\
&+(a_4+a_{2n-4})\left(\frac{n^{(2)}}{\lfloor 2}q_0+(n-2)q_2+q_4\right)\\
&+(a_{2s}+a_{2n-2s})\left(\frac{n^{(s)}}{\lfloor s}q_0+\frac{(n-2)^{(s-1)}}{\lfloor s-1}q_2+\ldots+(n-2s+2)q_{2s-2}+q_{2s}\right)\\
&+\ldots\\
&+(a_1+a_{2n-1})q_1\\
&+(a_3+a_{2n-3})((n-1)q_1+q_3)\\
&+(a_5+a_{2n-5})\left(\frac{(n-1)^{(2)}}{\lfloor 2}q_1+(n-3)q_3+q_5\right)\\
&+\ldots\\
&+(a_{2s+1}+a_{2n-2s-1})\left(\frac{(n-1)^{(s)}}{\lfloor s}q_1+\frac{(n-3)^{(s-1)}}{\lfloor s-1}q_3+\ldots+(n-2s+1)q_{2s-1}+q_{2s+1}\right)\\
&+\ldots\\
=&\,q_0\left(a_0+a_{2n}+n(a_2+a_{2n-2})+\frac{n^{(2)}}{\lfloor 2}(a_4+a_{2n-4})+\ldots\right)\\
&+q_1\left(a_1+a_{2n-1}+(n-1)(a_3+a_{2n-3})+\frac{(n-1)^{(2)}}{\lfloor 2}(a_5+a_{2n-5})+\ldots\right)\\
&+q_2\left(a_2+a_{2n-2}+(n-2)(a_4+a_{2n-4})+\frac{(n-2)^{(2)}}{\lfloor 2}(a_6+a_{2n-6})\ldots\right)\\
&+\ldots\\
&+q_{2s}\left(a_{2s}+a_{2n-2s}+(n-2s)(a_{2s+2}+a_{2n-2s-2})+\frac{(n-2s)^{(2)}}{\lfloor 2}(a_{2s+4}+a_{2n-2s-4})\ldots\right)\\
&+q_{2s+1}(a_{2s+1}+a_{2n-2s-1}+(n-2s-1)(a_{2s+3}+a_{2n-2s-3})+\ldots)\\
&+\ldots\\
=&\,q_0\mathrm{E}^n a_n+q_1\mathrm{E}^{n-1}a_n+q_2\mathrm{E}^{n-2}a_n+\ldots\ q_{2s}\mathrm{E}^{n-2s}a_n+q_{2s+1}\mathrm{E}^{n-2s-1}a_n+\ldots\ q_n a_n.
\end{aligned}$$

University of London,
King's College:
1914 *April* 25.

The Sun's Place within the Star-sphere. By O. R. Walkey.

The solution of this somewhat elusive, and consequently attractive, problem is being attempted by several methods, direct and indirect. A promising line of study is that of the relative star density in different parts of the galactic sphere, and one which, both direct and simple, requires (in view of our present data) no great mathematical refinement.

Professor Seeliger, some years ago, determined the density of the ninth magnitude stars with reference to galactic latitude, in which the increasing density towards the galactic plane argues a somewhat elliptic form about the galactic axis. These densities, as corrected in the late Professor Newcomb's "The Stars," indicate, for these ninth magnitude stars, a slight displacement of our system northward of the galactic plane—the resultant dip of the true equator being 1°·97 southward. This northward displacement is amply confirmed by Newcomb's detailed study * wherein he summarises (p. 18): "It therefore appears that this [the galactic] stream is not, on the whole, a great circle, but has a deviation showing a small, but well-marked, displacement of our system from the central plane in the direction of Coma Berenices in which the north galactic pole is situate."

The amount of this deviation, derived from Newcomb's figures for the main stream, was 1°·75, which is in striking agreement with that deduced from Seeliger's study. The fact of such a southward deviation is again confirmed by Professor Pickering in vol. lvi. of the *Harvard Annals.*

Accepting, therefore, as a fact our displacement northward of the galactic plane, the question next arising is whether there be any corresponding lateral displacement parallel to this plane. This involves a study of star densities with reference to galactic longitude.

The only study, apparently, in this direction, is that given in *Harvard Annals*, vol. lvi. parts i. and ii., where, however, no "resultants" were derived.†

Here one may, perhaps for simplicity's sake, coin the words

* "Contributions to Stellar Statistics," No. 1 (*Carnegie Inst. Pub.*, No. 10), and based on 42 bright patches in the main galactic stream, and 5 in the "arm" leaving it in Cygnus.

† E. Hertzsprung (*A.N.*, 4600) has deduced from *H.A.*, lvi., the poles of the respective planes of concentration of the several spectral types there indicated, but gives no resultant in galactic longitude beyond stating the semicircles most favoured by the O and B types.

"galongitude" and "galatitude" to denote henceforth galactic longitude and latitude respectively. All the galongitudes are reckoned "eastward" from the intersection of the celestial and galactic equators in Aquila.

Taking N and n to represent the respective numbers of the stars (down to a given magnitude) appearing in any two equal and mutually antipodal areas, the resultant in the direction of N has been represented by the simple "moment" $N - n$.

Though the N, n differences may be due to causes ranging from a variable star-interval with equal star depth, to the converse, yet, considering the incompleteness of our present knowledge, the simple ratio propounded here will suffice, and, as a matter of fact, computations made on the other hypothesis affected the resultant direction but little.

As a test of the reality of a lateral preponderance, preliminary counts were taken of stars down to different magnitudes, irrespective of their spectral type, and referred to the great circle having its north pole in α 12^h 47^m, $\delta + 27°$. Resultants were derived (in galongitude) from the antipodal pairs of galactic fields treated individually, wherein the richer (N) fields, lying generally together, afforded consistent evidence of lateral displacement. In view of local inequalities, the sections were further combined into successive quadrants, and a second resultant derived. Both of these are given in Table I. summarising these counts, which were as follows:—

I. Stars down to the fourth magnitude (R.H.P. 4·05). These were found, on a galactic chart, to lie symmetrically about $-5°$ galatitude; the numbers in each 10° zone of the 50° galactic belt chosen being 45, 50, 81, 55, and 46, north to south. The whole belt was divided into 16 sections $22\frac{1}{2}°$ long, and resultants derived for it and its middle zone alone.

II. All the stars of *H.A.*, l. (*i.e.* down to 6·50 R.H.P.) appearing in 24 patches, each 100 square degrees in area, lying at 15° intervals along the galactic equator.

III. The stars of the Bonn and Cape D.M.'s in the same 24 patches. It was necessary in this case to reduce the counts to a common basis, and herein lies the weakness of this and all other counts involving the D.M. stars, and due to the well-known inequalities in the limiting magnitudes. A comparison of Argelander's and Schönfeld's numbers in similarly situated areas within the galactic stream gave the ratio $\frac{\text{Schönfeld}}{\text{Argelander}} = 1{\cdot}22$.*

The standard adopted for the reduced counts was that of Schönfeld's numbers (corresponding to about 9·6 magnitude), the counts from Argelander being increased therefore in this ratio. To reduce the Cape D.M. numbers to the Schönfeld standard, a curve was plotted for star density from comparison of the intra-

* In areas outside the Galaxy the difference was rather more marked, ratio being 1·27, the mean ratio for all comparable areas being 1·25.

and extra-galactic portions of their overlapping belt 4° wide (in 19° to 23° S. declination).*

IV. The D.M. stars (reduced to the same standard) were next counted in a continuous belt 25° wide, extending from + 10° to − 15° galatitude, the further southern limit being chosen in view of the previously indicated southern deviation of maximum density; the belt was divided into 16 lengths of $22\frac{1}{2}°$ extent, and included 110,604 stars from Argelander, 33,966 from Schönfeld, and 235,065 from the Cape D.M. (of which 13,705 overlapped with Schönfeld). The major (N) sections, however, did not lie consistently together, the $(N - n)$ differences, moreover, being relatively small. In view of the uncertainties thus implied, the resultant of this count, taken by itself, would argue no more than a tendency towards the direction indicated.

V. The D.M. stars covering the whole sky, divided into 4 galactic quadrants and sub-quadrants, as indicating the effect, if any, of stars in high galatitudes. The totals appearing from each catalogue being: Argelander, 324,197; Schönfeld, 133,581; and Cape, 454,384, of which 33,762 overlapped with Schönfeld. The effect of the high galatitudinal stars is, allowing for the systematic uncertainty, negligible.

VI. Sixthly, as an index to the trend of yet fainter stars, a count was made of stars down to about the eleventh magnitude, appearing within the sections (each of 172 square degrees) of Celoria's 6-degree belt, intercepted within the 25-degree galactic belt of count IV.† These intercepts lie across the Galaxy in Aquila and Argo, their centres cutting the galactic equator in $3\frac{1}{2}°$ and $176\frac{1}{2}°$ galongitude. The Argo intercept was found to contain 25,820 stars as against 23,603 in the equal area in Aquila, giving a preponderating ratio 1·094 towards Argo. This, *so far as it goes*, confirms the previous findings; the ratio for the same intercepts with Argelander's B.D. stars being $\frac{4500}{3777} = 1·19$.

These six preliminary counts distinctly confirm M. Easton's opinion that the Cygnus region of the Galaxy lies nearer to us than that in Argo.

As a matter of information, the resultant was derived for the 42 bright patches given for the main galactic stream in Newcomb's earlier mentioned paper, and which was found to lie in galongitude 285°·3, while the inclusion of the five branch patches gave a direction 283°·5. The—in the present sense—arbitrary choice, however, of bright regions only, renders their resultant inadmissible for our purpose.

* The variations from the mean curve were considerable, the Cape D.M. density near the galactic south pole being actually less than that of Schönfeld, though the C.P.D. limit is nominally a magnitude fainter than that of the S.B.D. The resulting curve is therefore but approximate.

† Since Celoria's belt lies entirely to the north side (0° to 6°) of the celestial equator, its resolution into galactic quadrants gives a misleading resultant (50·4°), for the galactic semicircle 0° to 180° comprises stars nearer the Galaxy and consequently more numerous.

TABLE I.

No.	Limiting Mag.	Basis.	Area Sq. Deg.	No. of Stars involved.	Resultant Directions. Individually.	Resultant Directions. Groups.	Richest Semicircle in Galongitude.
I.	4·05	50° belt	17,368	277	243°·7	244°·2	157½ to 337½
		[10° mid-belt	3,582	81	259°·9	269°·9	202½ ,, 22½]
II.	6·50	24 patches	2,400	849	242°·8	240°·0	127½ ,, 307½
III.	9·6	24 patches	2,400	116,954	221°·5	221°·9	127½ ,, 307½
IV.	9·6	25° belt	8,920	369,635	264°·9	264°·1	157½ ,, 337½
V.	9·6	Whole sphere	41,253	912,162	...	264°·6	...
VI.	11·0	Celoria's belt	344	49,423	Towards Argo		...

The second row in Table I. is bracketed, since the number of fourth magnitude stars in the mid-zone is too few to give a true resultant. The wide discrepancy between the resultants in III. and IV. is doubtless due to the inequalities in reduction above mentioned for the D.M. stars.

Turning now from descending magnitude to spectral type as classified in *H.A.*, lvi. part i. (Table V.), we find the several types under the headings B, A, F, G, K, M, and "Peculiar," grouped into quadrants at 90°, 180°, 270°, and 360° galongitude, with reference to the circle having its north pole in α 12^h 40^m, $\delta + 28°$ —that adopted in all the *H.A.* tables. These quadrants are divided into five galatitude zones at 10°, 20°, 30°, and 60° on either side of the galactic plane. The number in the galatitudes higher than ± 10° have been resolved parallel to their equator (*i.e.* reduced by the cosine of the galatitude of their respective centres of distribution). The resultant in galongitude for each type is presented in Table II.*

TABLE II.

Type	B.	A.	F.	G.	K.	M.	Pec.	All.
No. of Stars	391	16,337	4886	2110	6543	356	113	30,736
Resultant	237°·3	250°·4	234°·7	239°·9	243°·7	251°·0	274°·5	246°·0

The figures of Table II., though merely indicative on account of the wide (90°) grouping, are none the less instructive as bearing out the previously derived resultants. The tendency of type A (as well, so far as their numbers imply, as of M and "Peculiar") seems to be farther "eastward" than the others; this divergence of type A, as marked in galongitude as it is in galatitude (*vide H.A.*, lvi.), is curious and may be due to the presence of these stars both near to as well as remote from our system—the nearer stars appearing to preponderate more easterly in galongitude than do those more remote.†

In order to gain a true idea of our position in the sidereal

* The polar (±60 to ±90 galatitude) zones were omitted in each case, owing to the omission of these for the third quadrant in the *H.A.* Table.

† This tendency finds support from the appearance, in a parallax chart of the Galaxy, of the nearer stars to segregate farther east in galongitude.

universe, it becomes necessary to eliminate the undue local effect of our neighbours, and this will be simplest done by confining ourselves to the B (Orion) type stars, as standing clear of such local influence; this is further justified by the absence in this type generally of preferential motion (Professor Campbell, *L.O.B.*, 195; B. & L. Boss, *A.J.*, 620, 635–6). Detailed counts of this class appear in *H.A.*, lvi., part ii. (Table II.), dividing the galactic sphere into meridians 20° apart and into zones at 10°, 20°, and 30° each side of the galactic plane. The corrections in the Appendix (Tables IV. and V.) bring the numbers of the "General" (Oe5 to B9) stars up to 834, and the "Well Marked" (Oe5 to B5) to 706. The numbers in the higher galatitudinal zones were resolved, as before, parallel to their equator.

As in the earlier counts, so here, the sections were grouped into quadrants and their intermediates. Triple sets, too, of sextantal (60°) groups were also taken, their respective "resultants" being presented in Table III. for the "General" and "Well Marked" classes; the major (N) sections in either instance extended from 160° to 340° in galongitude.

TABLE III. (Type B).

Resultant by:—	Individual Sections.	Quadrants.	Sextants.	Mean by Grouping.	Final Mean.
General (834 stars) . .	238°·4	238°·7	239°·2	239°·0	238°·7
Well Marked (706 stars) .	234·8	234·6	235·0	234·8	234·8

The figures in Table III. are in striking agreement, it may be observed, with those for type B (quadrants) in Table II. The consistent divergence of the "General" some 4° "Eastward" of the "Well Marked" evidently indicates the effect of including the B8–9 stars, which properly belong to class A and partake of the more eastward preponderance of this class. There is, however, one source of error which must not be overlooked, and that is the effect of a larger proportion of faint stars being "recognised" in the southern hemisphere than in the northern, and due in some measure to the clearer atmosphere at Arequipa, where the southern observations were taken.*

The effect of this inequality will be to throw the resultant unduly towards the 270° galactic meridian, *i.e.* east of the true direction. The amount of this error was roughly determined from a comparison with a count excluding all stars fainter than 6·50 magnitude, seeing that the remaining bright stars (which appear in *H.A.*, vol. l.) are complete for both hemispheres.†

* Professor Pickering in a letter says: "The fact that fainter southern stars are given is probably due to the greater clearness of the sky at Arequipa."

† A count of the "General" type B stars fainter than magnitude 6·50 north and south of −25° declination—the line evidently dividing the Harvard and Arequipa zones—gave 105 south as against 55 north of this line, leaving an actual ratio $\frac{\text{Arequipa}}{\text{Harvard}} = \frac{360}{314}$ for the brighter stars.

The effect of this exclusion was to throw the resultant about a degree "westward," whence for the present we may adopt 238° and 234° as the resultant directions of the "General" and "Well Marked" stars respectively of type B. Considering the features of the B8–9 stars in common with type A, a truer indication is given by the "Well Marked." The trend of further observations of these B stars, given in parts iv. and v. of *H.A.*, vol. lvi. (nearly doubling the numbers), appears still farther westward, and so we may provisionally place the sidereal centre as lying approximately on the 230° galactic meridian.

The position on this meridian of the sidereal centre as viewed from our northward displacement will naturally appear south in galatitude, the amount of this southward dip depending on the interrelation of our vertical and lateral displacements. This can be determined simply, and with accuracy sufficient for our present data, by a comparison of the numbers of (B) stars appearing respectively north and south of the apparent galactic equator, as an index to our relative vertical displacement, while a similar comparison of the richest meridional (galactic) hemisphere (that containing therefore the centre) with that more sparse (the "outer" hemisphere) will indicate our relative lateral displacement. The mutual intersection of the two planes thus derived (*i.e.* parallel to the sidereal equator and polar axis respectively), with their common normal through the centre, will define our relative place in the star volume.

A reasonable assumption must here be made of the form of the universe represented by the B stars. Plotting the cube roots of the numbers of these stars according to galatitude indicates a vertical elliptic section with a minor (*i.e.* polar) axis about half the major (equatorial)—allowing for local inequalities; while a similar treatment of the numbers in galongitude gives (as might be expected) a circular plan. The resultant form would therefore appear as an oblate spheroid of polar axis about half the equatorial.*

It is in the uncertainty of this ratio that the weakness of the deduction will mainly lie. The numbers indicate (for an oblate spheroid) a displacement vertically 0·271 of the polar radius and laterally 0·368 of the equatorial radius, which, allowing for the uncertainty named, will place the sidereal centre at an apparent dip of from 20° to nearly 30° south in galatitude, or on a line extending from α 6^h 0^m, $\delta - 55°$ to 6^h 55^m, $\delta - 52°$, and symmetrically, adjacent to the bright star Canopus.

The comparatively small number (706) of B stars used leaves the deduction open to further confirmation. Nearly twice this number is available in the later parts, *H.A.*, lvi., but the once intended work of reducing these to their galactic co-ordinates has been postponed in view of the several thousand stars of this class

* The fact that the B stars may lie in a disc form would tend to raise our relative vertical displacement, and this slight effect has been allowed for in the resultant dip angle.

now being classified at Harvard, a similar treatment of which will afford valuable evidence for or against the present deductions. The fact, however, that Professor Campbell finds (*L.O.B.*, 195) that the brighter B stars (all of which figure in the present result) lie as far scattered as those fainter will lessen the uncertainty of our provisional result, while any symmetrical increase around the star form here adopted will not affect the dip angle indicated.

Finally, as to the probable distance of the sidereal (B star) centre, an indication is supplied by the studies of Kapteyn, Boss, and Campbell, who find for the 400 B stars (using round numbers) a mean distance of 500 light-years. The actual "equatorial" radius of their containing spheroid would be anything greater, and should be fairly indicated by Campbell's estimate of 50 per cent. or more increase, say to 800 l.y. The corresponding radius containing the 706 stars of the present study should be (in spite of the relative remoteness of the bright stars included in the smaller number) yet larger, and approach 1000 l.y., wherein our relative eccentricity here derived would place us some 400 l.y. from the centre. Thus the apparent dip of the centre, south in galatitude, would combine with that of the galactic main stream to place the latter at a distance of some 5000 l.y. in Cygnus and 6000 l.y. in Argo.

The Double Star OΣ 137. By W. S. Franks.

OΣ 137 *rej.* (P. VI 62) R.A. $6^h\ 15{\cdot}3^m$: Decl. $+21°\ 11'$ (1900). On coming across the wide pair AC (Berlin Zone 2324, 7·7 mag.; and 2322, 9·1 mag.), which is also S 513 and OΣΣ 76, I had some difficulty with Hussey's description (*Lick Observatory Publications*, vol. v.), which ran A = 7·7, B = 9·1, C = 8·0 mag.—where AB = 258°·1 : 59″·75 and AC = 83°·7 : 15″·22 (epoch 1899·01); and quoting prior measures by Mädler. All I could see was the wide pair OΣΣ 76, described by Dembowski, who gave 7·2, 8·5 : 257°·5 : 59″·45 : 1872·30 : 3*n*. I estimated the mags. as 7·5 and 8·5, and my measures on two nights were in close agreement with his. I also measured the only other bright star in the field, which I estimated at 7·7 mag., at 23°·0 : 265″·16 from A. I failed to see B—erroneously called 9·1 mag. Afterwards I came across De.'s record of a small star near C, which he saw, but could not measure. My previous measures having been made in strong twilight, I devoted some time on the next fine night (May 2) with a dark sky, save a little moonlight, to a careful re-examination of the group. With close attention I detected a small star (D), which I estimated at 11·0 mag. (Σ's scale) and roughly 320° : 35″ ± from C—thinking this might be what De. saw (but probably it was the one that Mädler

measured). The instrument used was a $6\frac{1}{8}$-in. Cooke refractor, and power 235 on micrometer. The measures are as follows:—

AC	257°·2	58″·91	1825·11	South.
	256·7	60·29	1843·24	Mädler.
	257·5	59·45	1872·30	Dembowski.
	257·6	59·28	1893·86	Franz.
	258·1	59·75	1899·01	Hussey.
	257·9	59·20	1905·13	Burnham.
	257·4	59·20	1914·32	Franks.
	*258·2	60·07	1914·37	Bowyer.
CB	66·7	16·28	1843·24	Mädler (B = 10).
	83·7	15·22	1899·01	Hussey . . .
	85·3	14·75	1905·09	Burnham (= 12).
	*86·6	14·85	1914·37	Bowyer (= 11·5).
CD (Fr.)	*325·3	41·53	1914·37	Bowyer (D = 10·5).

Hussey pointed out that the small star B showed proper motion, which is confirmed by the later measures, and amounts to ″·09 in direction 181°. It would be interesting to note if the other small star D participates in this motion. AC are relatively fixed, though there may be a little common proper motion.

* These measures were made with the 28-inch Greenwich refractor, through the courtesy of Mr. Lewis, to whom I am indebted for other valuable information.

Mr. Frederick J. Hanbury's Observatory:
Brockhurst, East Grinstead.